

I f G f

☆



$$A_N = \sum_{\xi \in \mathbb{Z}} \hat{A}(\xi) f(\xi) \quad (8)$$

$$A = \sum_{\xi \in \mathbb{Z}} \hat{A}(\xi) f(\xi) \quad (1)$$

$$W = \delta(\xi - \xi'), \quad (4)$$

$$F = \sum_{\sigma} f(2\pi\sigma)$$

$$U = \sum_{\xi} \hat{F}(\xi) \quad (3)$$

$$G = \sum_{\xi} \frac{\pi}{\xi} \int_{-\infty}^{\infty} \frac{\omega^2}{4} \omega \quad (3)$$

$$G = \sum_{\xi} \frac{\pi}{\xi} \int_{-\infty}^{\infty} \frac{\pi^2 \omega^2}{2\pi\omega} 2\pi\omega \quad (2)$$

$$C = \sum_{\xi} \hat{G}(\xi) \quad (8)$$

$$G = \sum_{\xi} \frac{\pi}{\xi} \int_{-\infty}^{\infty} \frac{\pi^2 \omega^2}{2\pi\omega} 2\pi\omega \quad (8)$$

$$G = \sum_{\xi} \frac{\pi}{\xi} \int_{-\infty}^{\infty} \frac{\pi^2 \omega^2}{2\pi\omega} 2\pi\omega \quad (8)$$

$$\Gamma = \sum_{\xi} \hat{G}(\xi) \quad (8)$$

$$\Gamma = \sum_{\xi} \hat{G}(\xi) \quad (8)$$

$$\Gamma = \sum_{\xi} \hat{G}(\xi) \quad (8)$$

$$\xi < \frac{\epsilon}{2\pi} \quad \frac{\epsilon}{\pi} \quad (9)$$

$$W = \sum_{\lambda \in \mathbb{Z}} \hat{W}(\lambda) \quad (4)$$

$$W = \sum_{\lambda \in \mathbb{Z}} \hat{W}(\lambda) \quad (4)$$

$$U = \sum_{\xi} \hat{U}(\xi) \quad (3)$$

$$\hat{\xi} \approx \frac{1}{\lambda} \quad \gamma_{\lambda} v \xi \quad (11)$$

$$g_n = \sum_{\xi} \hat{g}_n(\xi) \quad (26)$$

$$N = 2, \quad v \quad (11)$$

$$A = \sum_{\xi} \hat{A}(\xi) \quad (8)$$

$$G = \sum_{\xi} \hat{G}(\xi) \quad (12)$$

$$G = \sum_{\xi} \hat{G}(\xi) \quad (12)$$

$$C = \sum_{\xi} \hat{C}(\xi) \quad (12)$$

$$\hat{G} = \sum_{\xi} \frac{\pi}{\xi} \int_{-\infty}^{\infty} \frac{\pi^2 \omega^2}{2\pi\omega} \frac{\gamma_{\lambda} v \frac{\xi}{\pi} \omega}{\lambda \frac{\xi/\pi \omega}{vN}} \omega \quad (12)$$

$\xi < \frac{1}{4}$ G A f

$$\frac{\pi}{\omega} \frac{\pi^2 \omega^2}{2\pi \omega} \lambda v^2 \frac{\omega}{\pi} \frac{\pi \omega^2}{v N^2 \lambda} \omega$$

$$\frac{vN}{\kappa} \lambda \pi \times \frac{\lambda^2 v^2 N^2 \pi \omega^2 \lambda^2 \pi^2 \lambda v^2 N^2 \omega^2}{\kappa}$$

$$\times 2 \frac{\pi^2 \lambda^2 v^4 N^2 \pi \lambda^2 v^3 N^2}{\kappa}$$

$$\kappa \pi^2 \lambda v^2 N^2 \pi^2 \lambda^2 v^4 N^2 \quad 14$$

Γ

$$\tilde{G} \frac{vN}{\kappa} \lambda \pi \frac{\pi^2 \lambda v^2 N^2 \omega^2 \lambda^2 \pi^2 \lambda^2 v^4 N^2}{\kappa}$$

$$\times \tilde{\Gamma} \frac{\lambda^2 v^2 N^2 \pi \omega^2}{\kappa} \frac{2 \pi \lambda^2 v^3 N^2}{\kappa} \quad 1$$

$$\tilde{\Gamma} \frac{\lambda \pi^2 \omega^2}{\kappa} \quad 16$$

Γ f (3) (1) f
 4 199 84 6 1 9626 9 9626 81 2283491 1 6 1 4 4 (-) 2 31 (433) -3331 (138 064434 3 -E

$$M \left(f, \begin{matrix} \textcircled{a} \\ f \end{matrix}, \begin{matrix} \textcircled{a} \\ f \end{matrix} \right) \leq \epsilon,$$

$$\left(\begin{matrix} \textcircled{a} \\ f \end{matrix}, \frac{1}{2} \right) \leq \frac{\epsilon}{2}, \quad \left(\begin{matrix} \textcircled{a} \\ f \end{matrix}, \begin{matrix} \textcircled{a} \\ f \end{matrix} \right) \leq \frac{\epsilon}{2}.$$

$$C_k \leq \frac{\epsilon}{4}.$$

$$\Gamma(f) \leq 4 \left(\frac{\epsilon}{2}, \frac{\epsilon}{2} \right) \leq 4 \frac{\epsilon}{2} = 2\epsilon.$$

\mathbb{W} f $\frac{a}{z}$ f \mathbb{W} $\frac{a}{z}$ f FF $\frac{a}{z}$ f
 \mathbb{W} f $\frac{a}{z}$ f f $\frac{a}{z}$ f f f
 $N = 12$ M fFF f $\frac{a}{z}$ f $\frac{a}{z}$ f 3 FF
 f $N = 128$ FF f $N = 8192$ $\frac{a}{z}$ f 3 FF
 F f 1 $\frac{a}{z}$ 3 $-$
 f 1 $a < 138$ f f 1 $-$
 f

5. Conclusions

\mathbb{W} -8 8 \mathbb{W} $8-44$ $(\mathbb{W})-239$ (8) $3 f$,

@

[

@f

F

M f

$r_{\lambda} \xi$ $\hat{r}_{\lambda} \xi$, 2 ,
 $\in \mathbb{Z}$

A .4

A f_{ξ}^{ν} , f_{ξ}^{ν} **A 9)** $\hat{\xi}$

$$\mathcal{P}^{\nu} = \int_{-\infty}^{\infty} 2\pi \xi^{\wedge} \nu N \xi \hat{\gamma}_{\lambda} \xi \xi, \quad \mathbf{A 10}$$

$$\mathcal{P}^{\nu} = \int_{-\frac{1}{2}}^{\frac{1}{2}} 2\pi \xi \hat{\nu} N \xi \hat{\gamma}_{\lambda} \xi \xi, \quad \mathbf{A 11}$$

W $\hat{\xi}$, $\hat{\gamma}_{\lambda} \xi$ **A 10)** **A 11)**, f , $f \lambda$, $1/\nu$

$$F \xi = \int_{\mathbb{Z}} \mathcal{P}^{\nu} \xi \xi, \quad \mathbf{A 12}$$

$$F \xi = \int_{\mathbb{Z}} \hat{\nu} N \xi \hat{\gamma}_{\lambda} \xi \xi, \quad \mathbf{A 13}$$

$\hat{\xi}$ **A 13)** $\hat{\xi}$

Then

$$E_\infty \leq 1 - \hat{\phi}(\alpha) \frac{1}{C^{\alpha, \alpha}}, \quad \alpha \in \mathbb{Z}^+, \quad \alpha \hat{\phi}(\alpha) \leq \alpha.$$

$$\hat{\phi}(\alpha) = \sum_{l=1}^{\alpha} \frac{f(\hat{\phi}(l))}{l} \quad \text{for } \alpha \geq 1, \quad \hat{\phi}(0) = 0.$$

F, $\hat{\phi}(\xi) \pm 1$, $\lambda < 1.4$ F, $\lambda = 1.4$ F, $\lambda = 1.4$ F, $\alpha \frac{1}{2v}$.

$$\hat{\phi}(\xi) = \sum_{l=1}^{\xi} \frac{f(\hat{\phi}(l))}{l}, \quad \xi \leq \frac{N}{2}, \quad \xi < \frac{1}{4}, \quad \hat{\phi}(\xi) \leq \xi.$$

A 9)

f33

(2) C

$$F(\xi) = \int_{-\frac{vN}{2}}^{\frac{vN}{2}-1} \mathcal{P}^v e^{-2\pi i \xi t} dt$$

A 16

FFI

(3) $\rightarrow F(\xi_j) = \dots, \lambda \xi$

A.2.2. Fast evaluation of the Fourier series at unequally spaced points

$$L \int_{-\infty}^{\infty} f(t) e^{-2\pi i \xi t} dt = \sum_{n=-\infty}^{\infty} a_n(\xi) e^{-2\pi i \xi t_n}$$

$$\xi = \frac{F(\xi/v)}{\dots, \lambda \xi/v}$$

$$F(\xi) = \dots \mathbf{A} \ 1 \ 2) \quad a_n(\xi) \ \mathbf{A} \ 4) \ C$$

$$\hat{f}(\xi) = \dots \int_{-\infty}^{\infty} f(t) e^{-2\pi i \xi t} dt$$

$$f(\xi) = \sum_{n=-\infty}^{\infty} \hat{G} \frac{\xi}{v} \tilde{\mathcal{P}}^v \xi, \tag{A.20}$$

$$\hat{G} \xi = \frac{1}{\sqrt{v}} \frac{e^{-i \frac{2\pi v \xi}{vN}}}{e^{-i \frac{2\pi v \xi}{vN}}}, \tag{A.21}$$

(A.19) (A.20)

$$f(\xi) = \sum_{n \in \mathbb{Z}} \hat{G} \frac{\xi}{v} \gamma_\lambda vN \tilde{\mathcal{P}}^v \xi = N \hat{G} \xi, \tag{A.22}$$

$$\hat{G} \xi = \sum_{n \in \mathbb{Z}} \hat{G} \frac{\xi}{v} \gamma_\lambda v \xi. \tag{A.23}$$

$$N \hat{G} \xi = \sum_{n \in \mathbb{Z}} \frac{1}{\sqrt{v}} \frac{e^{-i \frac{2\pi v \xi}{vN}}}{e^{-i \frac{2\pi v \xi}{vN}}}, \quad \frac{N}{2} \leq \xi \leq \frac{N}{2} + 1; \tag{A.24}$$

$$A \text{ FFT } f = \sum_{n \in \mathbb{Z}} \hat{G} \frac{vN}{v} f = \sum_{n \in \mathbb{Z}} \hat{G} v f.$$

Algorithm 2.

- (1) C
- (2) A FFT $\hat{G} \xi$ (A.24)
- (3) C $\hat{G} \xi$ (A.23)

A.2.3. Evaluation of unequally spaced FFT at unequally spaced points

$$f(\xi) = \sum_{n \in \mathbb{Z}} \hat{G} \frac{\xi}{v} \gamma_\lambda vN \tilde{\mathcal{P}}^v \xi, \quad \xi \in \left\{ \frac{N}{2}, \frac{N}{2} + 1, \dots, \frac{N}{2} + N \right\} \tag{A.14}$$

Algorithm 3.

- (1) C
- (2) A $\tilde{\mathcal{P}}^v$ $\gamma_\lambda vN \xi$ $\frac{v^2 N}{2}, \dots, \frac{v^2 N}{2} + 1$ (A.2)
- (2) A $\hat{G} \xi$ ξ $\frac{v}{2}, \frac{v}{2} + 1, \dots, \frac{v}{2} + N$

