

1. (24 points) The following problems are not related. If a limit does not exist, you must say so. If you use a theorem, clearly state its name and show that its hypotheses are satisfied.

(Reminder: You may not use L'Hôpital's Rule or "Dominance of Powers" in any solutions on this exam.)

(a)  $\lim_{x \rightarrow 0} \frac{\sec x}{4x \cot 2x}$

(b)  $\lim_{x \rightarrow 1} \frac{\sin^2 x}{x}$

(c)  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{5 - x^2}$

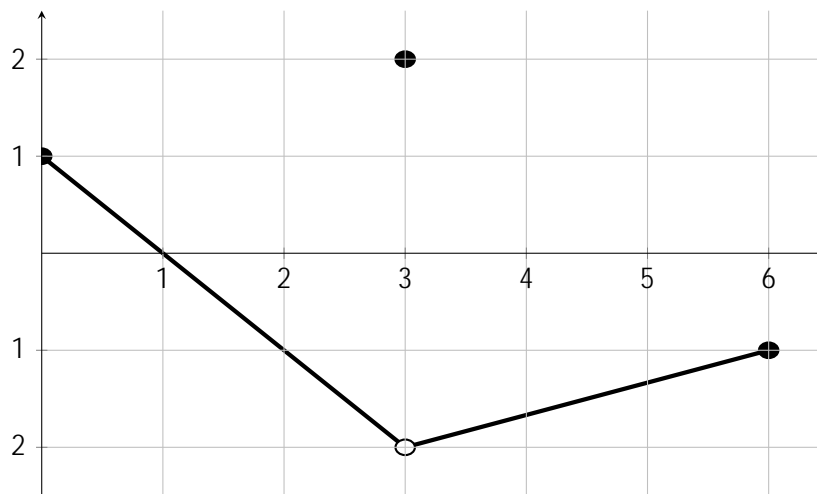
2. (21 points) The following problems are unrelated.

(a) Given that  $\csc \theta = \sqrt{5}$  and  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ , find the values of  $\tan \theta$  and  $\cos(2\theta)$ .

(b) Find all values of  $x$  in the interval  $[0; \pi]$  that satisfy  $\tan x \sec x = 4 \sin x$ .

(c) A squirrel is up a tree, and it sees a peanut on the ground some distance away. If the straight-line distance between the peanut and the squirrel is 50 ft, and the angle between the straight-line and the tree is  $\theta = 6$  radians, how far down the tree and across the ground must the squirrel travel to reach the peanut? Give your answer with appropriate units.

3. (15 points) Shown below is a graph of  $y = f(x)$ , which consists of two line segments with a single removable discontinuity.



(a) Find a formula for  $f(x)$ .

(b) Sketch a graph of  $y = f(x) + 1$ . Label the intercepts, if any.

(c) Suppose we use the precise definition of a limit to verify the value of  $\lim_{x \rightarrow 3} f(x)$ , and we find that if  $4 < x < 6$ , then  $\frac{5}{3} < f(x) < 1$ . What are the corresponding values of  $\epsilon$  and  $\delta$ ? (recall the precise definition of a limit: the limit of  $f(x)$  as  $x$  approaches  $a$  is  $L$  if for every number  $\epsilon > 0$ , there is a corresponding  $\delta > 0$  such that if  $0 < |x - a| < \delta$ , then  $|f(x) - L| < \epsilon$ .)

4. (20 points) Consider the function  $g(x) = \frac{2x^2}{x^2} \frac{12x + 16}{7x + 12}$ .