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3. (20 pts) Do the following integrals converge or diverge? Evaluate the convergent integrals.

(a)  $\int_0^1 \frac{2e^{2x}}{1+e^{4x}} dx$

(b)  $\int_2^1 \frac{1+\cos^2 x}{x-1} dx$

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(a) By direct calculation, we have

$$\begin{aligned} \int_0^1 \frac{2e^{2x}}{1+e^{4x}} dx &= \lim_{t \rightarrow 1^-} \int_0^t \frac{2e^{2x}}{1+(e^{2x})^2} dx && u = e^{2x}; du = 2e^{2x} dx \\ &= \lim_{t \rightarrow 1^-} \int_{e^{2 \cdot 0}}^{e^{2t}} \frac{1}{1+u^2} du \\ &= \lim_{t \rightarrow 1^-} \arctan(u) \Big|_1^{e^{2t}} \\ &= \lim_{t \rightarrow 1^-} \arctan(e^{2t}) - \frac{\pi}{4} \\ &= \boxed{\frac{\pi}{4}} \end{aligned}$$

Since the limit is finite, the integral converges.

(b) We suspect that this integral diverges (since we are dividing by  $x$ ) so let's try to show that via the comparison test. We can find a useful inequality as

$$\frac{1+\cos^2 x}{x-1} \geq \frac{1}{x-1} \geq \frac{1}{x}$$

Since  $\int_2^1 \frac{1}{x} dx$  is a divergent  $p$ -integral (i.e.  $p = 1 < 1$ ), the Comparison test tells us that  $\int_2^1 \frac{1+\cos^2 x}{x-1} dx$  is also divergent.

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4. (20 pts) Let  $R$  be the region bounded by  $y = 1 - x^2$  and  $y = x + 1$ .
- Sketch and shade the region  $R$ . Label all axes, curves, and intersection points.
  - Set up, but do not evaluate, integrals to determine each of the following:
    - The area of  $R$  using integration with respect to  $x$ .
    - The area of  $R$  using integration with respect to  $y$ .

Solution:

- Graphing our equations, intersection points, and shading  $R$ , gives us

(b) Using the graph, our integrals are as follows:

- From the graph, our integral is

$$A = \int_{-1}^1 (1 - x^2) - (x + 1) dx:$$

- Rewriting our equations in terms of  $y$  gives us  $x = \sqrt{1 - y}$  and  $x = y + 1$ . Then, using our graph, we can compute the area as

$$A = \int_0^1 (y + 1) - (\sqrt{1 - y}) dy + \int_0^1 \sqrt{1 - y} - (y + 1) dy:$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos 2x) \quad \sin^2(x) = \frac{1}{2}(1 - \cos 2x) \quad \sin 2x = 2 \sin x \cos x \quad \cos 2x = \cos^2 x - \sin^2 x$$

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$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C; u^2 < a^2$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C; u^2 > a^2$$

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$$\int_a^b f(x) dx \approx \frac{b-a}{n} [f(x_0) + f(x_1) + \dots + f(x_n)]; \quad x = \frac{b-a}{n}; \quad \bar{x}_i = \frac{x_{i-1} + x_i}{2}; \quad jE_{Mj} \quad \frac{K(b-a)^3}{24n^2}$$

h` T 2 x Q B / H \_ m H 2

$$\int_a^b f(x) dx \approx \frac{b-a}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]; \quad x = \frac{b-a}{n}; \quad jE_{Tj} \quad \frac{K(b-a)^3}{12n^2}$$