

APPM 2350—Exam 2

Wednesday Oct 20th, 6:30pm-8pm 2021

This exam has 4 problems. Please start each new problem at the top of a new page in your blue book. Show all your work in your blue book and simplify your answers. Answers with no justification will receive no

Problem 2 (30 points)

Given

$$U(x, y) = 1 + xy, \text{ where } x > 0 \text{ and } y > 0$$

- (a) Give the equation of the level curve of $U(x, y)$ that passes through the point $(1, 2)$.
- (b) Sketch the level curve you found in part (a). Label the value of

- (a) Find and classify all critical points of $T(x, y)$.
- (b) If you start at the location $(x, y) = (1, 0)$ and you move along a straight path toward the point
(

(a) The quadratic Taylor approximation of $G(x, y)$ centered at the point $(1, 0)$ is given by:

$$Q(x, y) = G(1, 0) + G_x(1, 0)(x - 1) + G_y(1, 0)y + \frac{1}{2}(G_{xx}(1, 0)(x - 1)^2 + 2G_{xy}(1, 0)(x - 1)y + G_{yy}(1, 0)y^2)$$

$$= -23 - 2(x - 1) - 5y + 2(x - 1)^2 + 4y^2 - 3(x - 1)y$$

Thus

$$\begin{aligned} G(3, -1) &= Q(3, -1) \\ &= -23 - 2(2) - 5(-1) + 2(2)^2 + 4(-1)^2 - 3(2)(-1) \end{aligned}$$

$$= -4$$

(b) Taylor's error formula for a quadratic approximation is given by:

$$|E(x, y)| \leq \frac{M}{3!} |x - 1| + |y|^3$$

where M is an upper bound on the function's 3rd order mixed partial derivatives on the region.

We are given

$$M = \frac{1}{4}, \quad |y| = 1.5 \quad \text{and} \quad |E(x, y)| \leq \frac{1}{3}$$

Thus

$$\begin{aligned} &\frac{1}{24} |x - 1| + 1.5^3 \leq \frac{1}{3} \\ &= |x - 1| + 1.5^3 \leq 8 \\ &= |x - 1| + 1.5 \leq 2 \\ &= |x - 1| \leq 0.5 \end{aligned}$$