This exam is worth 150 points and has 6 problems.

Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.

Please begin each problem on a new page.

DO NOT leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.

You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5” × 11” crib sheet with writing on two sides.

Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: ”I will abide by the CU Boulder Honor Code on this exam.” **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/072823 (20 pts)] A jellyfish swims along the path $\mathbf{r}(t) = t^2\,\mathbf{i} + \mathbf{j} - e^t\,\mathbf{k}$ from $0 \le t \le 3$ catching plankton as it moves along. If the density of plankton in the water is given by $P(x, y, z) = y\sqrt{4x + z^2}$ g/m find the total amount of plankton the jellyfish caught. Include units in your final answer.

2. [2350/072823 (30 pts)] The force of the current in Penelope the platypus’s river can be described by the vector field $\mathbf{F} = (2x + \tan y)\,\mathbf{i} + (x \sec^2 y)\,\mathbf{j}$. Penelope swims first along the curve C_1 to the base of a waterfall then later returns along the curve C_2 with

   C_1: the line segment from $(2, 1)$ to $(2, -1)$ $\qquad$ C_2: the curve $x = 2 + \cos\left(\frac{\pi}{2} y\right)$ from $(2, -1)$ to $(2, 1)$

   (a) (10 pts) Directly calculate the work done by the current on Penelope along C_1 by evaluating an appropriate line integral.

   (b) (10 pts) Find the potential function of $\mathbf{F}$.

   (c) (5 pts) Using a theorem from Calculus 3, determine the work done by the current on Penelope along C_2.

   (d) (5 pts) Determine the total work done by the current along the union of the two paths: $C = C_1 \cup C_2$.

3. [2350/072823 (36 pts)] Consider the open surface cut from $z = -3\sqrt{x^2 + y^2}$ where $-3 \le z \le 0$.

   (a) (5 pts) What quadric surface is this?

   (b) (10 pts) Give a parameterization of the boundary curve, C, of this surface with a counterclockwise orientation when viewed from above.

   (c) (5 pts) What portion of a plane shares the same boundary?

   (d) (16 pts) Use Stokes’ theorem to evaluate $\displaystyle\int_C 3yz\,dx + 7x\,dy + z\,dz$.

4. [2350/072823 (18 pts)] Consider the closed boundary C made by the curves $x = y^2$ and $x = -y^2 + 2$ oriented counterclockwise. Compute the flux of $\mathbf{H}$ through C if $\mathbf{H} = \langle e^{-y^2} + 3x^2, \ln x + 6 \rangle$.

5. [2350/072823 (22 pts)] Consider a three dimensional solid, E, bounded within $S_1 : x^2 + y^2 + z^2 = 4$ and below $S_2 : z = \sqrt{x^2 + y^2}$. Find the outward flux of $\mathbf{F} = \langle \sin y, x\ln(z + 1), z^2 \rangle$ through the boundary of E.

6. [2350/072823 (24 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) The function $f(x, y) = 1 - x^2 - y^2$ is guaranteed to have a minimum value for all x, y in the first octant.

   (b) The function $g(x, y) = e^{xy}$ has a saddle point at the origin.

   (c) The cross product of $\mathbf{k}$ and the acceleration vector of the path $\mathbf{r}(t) = 2t\,\mathbf{i} + 5t\,\mathbf{j} + t^2\,\mathbf{k}$ is never the zero vector.

   (d) The line whose vector equation is $\mathbf{r}(t) = \langle 1, 6, 2\rangle + t\langle -1, -3, 2\rangle$ intersects the xz-plane at the point $(-1, 0, 6)$.

   (e) $\displaystyle\lim_{(x,y)\to(-1,0)} \frac{xy + x}{(x + 1)^2 + y^2 - 2}$ does not exist.

   (f) The instantaneous rate of change of the function $g(x, y) = 4x^2 + 2x - 3y^2$ at the origin is largest in the $+\mathbf{j}$ direction.