



Check that the functions are linearly independent

$$W(x^3, x) = \begin{vmatrix} x^3 & x \\ 3x^2 & 1 \end{vmatrix} = 4x^3 \neq 0$$

so the two functions are linearly independent.  $x^3; x$  is a basis for the solution space.

(c) We need to use variation of parameters so start by putting the differential equation into standard form

$$y'' + \frac{3}{x}y' - \frac{3}{x^2}y = \frac{1}{x^3}$$

We'll let  $y_1 = x^3$  and  $y_2 = x$ ,  $f(x) = \frac{1}{x^3}$  from the differential equation and  $W[y_1; y_2] = 4x^3$  from part (b). The particular solution will have the form  $y_p = v_1 y_1 + v_2 y_2$  where

$$v_1' = \frac{y_2 f}{W[y_1; y_2]} = \frac{x \cdot \frac{1}{x^3}}{4x^3} = \frac{x}{4x^3} \Rightarrow v_1 = \int \frac{x}{4x^3} dx = -\frac{x^2}{8}$$

$$v_2' = \frac{y_1 f}{W[y_1; y_2]} = \frac{x^3 \cdot \frac{1}{x^3}}{4x^3} = \frac{1}{4x^3}$$

- (d) (3 pts) If  $\gamma = 3$  and the mass of the object is 4, what is the value of the spring/restoring constant if the oscillator is critically damped?

**SOLUTION:**

- (a)  $x(0) = 1; \dot{x}(0) = 0$   
 (b)  $\gamma = 0; \gamma_0 > 0; F_0 \neq 0; \gamma = \gamma_0$   
 (c) i. yes  
 ii. infinitely many  
 iii. yes  
 (d) Critically damped means  $4^2 - 4\frac{k}{m} = 0 \Rightarrow 9 - \frac{k}{4} = 0 \Rightarrow k = 36$

5. [2360/041322 (12 pts)] Characteristic equations for certain constant coefficient linear homogeneous differential equations are given along with a forcing function,  $f(t)$ . Give the form of the particular solution you would use to solve the nonhomogeneous [with the given  $f(t)$ ] differential equation from which the characteristic equation was derived using the Method of Undetermined Coefficients. Do not solve for the coefficients.

- (a) (4 pts)  $r(3r - 1) = 0; f(t) = 3 + \sin t$   
 (b) (4 pts)  $r^2 + 2r + 5 = [r - (1 + 2i)][r - (1 - 2i)] = 0; f(t) = e^{-t} + 5 \cos 2t$   
 (c) (4 pts)  $(r - 3)^3(r + 2) = 0; f(t) = te^{3t} + 2e^{-2t}$

**SOLUTION:**

- (a) Homogeneous solutions are  $1; e^{t/3}; y_p = At + B \cos t + C \sin t$   
 (b) Homogeneous solutions are  $e^{-t} \cos 2t; e^{-t} \sin 2t; y_p = Ae^{-t} + B \cos 2t + C \sin 2t$   
 (c) Homogeneous solutions are  $e^{3t}; te^{3t}; t^2e^{3t}; e^{-2t}; y_p = t^3(At + B)e^{3t} + Cte^{-2t}$