

Write your name below. You must show your work and not give decimal answers (i.e. don't use a calculator or software to compute a decimal answer). You are not allowed to collaborate on the exam or seek outside help, though using your notes, the book, the recorded lectures, or material

(h) Suppose \mathbf{A} is a symmetric matrix. Then, $\mathbf{Ax} = \mathbf{b}$ has a solution if and only if \mathbf{b} is orthogonal to the corange of \mathbf{A} .

(i) If \mathbf{A} is invertible, then \mathbf{A} is diagonalizable.

(j) The singular values of a nonsingular matrix \mathbf{A} are the same as the singular values of \mathbf{A}^{-1} .

2. Consider the following linear transformation: $L(x; y) = \begin{pmatrix} x + y \\ 2x - y \end{pmatrix}$.

(a) (4 points) Find the matrix form of $L(x; y)$ with respect to the standard basis.

(b) (6 points) Find the matrix form of $L(x; y)$ with respect to the following basis: $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. (The same basis is used for both the domain space and the co-domain space.)

3. (20 points) Find the Jordan decomposition of $\mathbf{A} = \begin{matrix} & & & & 2 \\ & & & & 3 \\ & & & 3 & 3 \\ & & 4 & 1 & 3 \\ & & & 1 & 1 \end{matrix}$

4. (20 points) Find the singular value decomposition of

$$\mathbf{A} = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \end{pmatrix} .$$

7. (10 points) Find the least-squares solution of the system

$$\begin{pmatrix} 2 & 1 & 1 & 0 \\ 6 & 1 & 0 & 1 \\ 6 & 0 & 1 & 1 \\ 4 & 1 & 1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 2 \\ 5 \\ 6 \end{pmatrix}$$