

APPM 3570 / STAT 3100

Fall 2021

Exam 01

September 29

GENERAL INSTRUCTIONS:

- This exam has two parts, and you may start on either as long as you follow the instructions for each.
- Notes, your text and other books, cell phones, and other electronic devices are not permitted, except for calculators.
- Calculators are permitted.
- Write your name and sign and date your pledge to the CU Honor Code in the box below.

On my honor as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on either part of this work.

Name (Last, First): _____

Signature: _____ Date: _____

ADDITIONAL INSTRUCTIONS FOR PART I: Use the last few pages of your bluebook as scratch paper to work out the following 10 questions. For each question, circle what you believe to be the correct answer. Ambiguous answers to a question will result in 0 points for that question. Do not justify your answers to these 10 questions.

1. (4 points.) What's the coefficient of z^0 (i.e. the constant term) in the expansion of

$$\left(z + \frac{1}{z}\right)^{20} ?$$

Solution: $\left(z + \frac{1}{z}\right)^{20} = \sum_{k=0}^{20} \binom{20}{k} z^k \left(\frac{1}{z}\right)^{20-k} = \sum_{k=0}^{20} \binom{20}{k} z^{2k-20}$. The coefficient of z^0 is associated with $k = 10$, which has the coefficient $\binom{20}{10}$.

2. (4 points.) Urn A has 4 green marbles and 3 red marbles. Urn B has 5 green marbles and 2 red marbles. A fair die is rolled. If the face shows a 2 then a marble is chosen at random from Urn A. Otherwise, a marble is chosen at random from Urn B. What is the probability that a green marble is selected?

Solution: Let T = the event that the die shows a 2. Let G be the event that a green ball is selected.

$$\begin{aligned} P(G) &= P(G|T)P(T) + P(G|T^c)P(T^c) \\ &= \frac{4}{7} \cdot \frac{1}{6} + \frac{5}{7} \cdot \frac{5}{6} \end{aligned}$$

$$= \frac{29}{42}:$$

3. (4 points.) The word "supercalifragilisticexpialidocious" means extraordinarily good, or wonderful. The letter count of this 34-letters word is given by the table below:

A	C	D	E	F	G	I	L	O	P	R	S	T	U	X
3	3	1	2	1	1	7	3	2	2	2	3	1	2	1

How many different words (in English or otherwise) may be constructed permuting the letters in "supercalifragilisticexpialidocious"?

Solution: Among the 34 possible positions, we need to select 3 to put A's, 3 to put C's, 1 to put a D, etc. The number of ways to do so is $\binom{34}{3, 3, 1, 2, 1, 1, 7, 3, 2, 2, 2, 3, 1, 2, 1}$

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8. (4 points.) 40% of students in a class wear a wristband of some type. 10% have a tattoo. 8% have both a wristband and a tattoo. What is the probability that a randomly selected person in the class has neither a wristband nor a tattoo?

Solution: $P(T^c W^c) = 1 - P(T \cup W) = 1 - (P(T) + P(W) - P(TW)) = 1 - (0.4 + 0.1 - 0.08) = 0.58$.

9. (4 points.) A discrete random variable X has probability mass function (p.m.f.):

$$p_X(x) = \begin{cases} c & ; \text{ if } x = \frac{1}{2}; \\ 3c & ; \text{ if } x = \frac{1}{5}; \\ 0 & ; \text{ otherwise;} \end{cases}$$

where c is certain constant. What value must c have?

Solution: Since X is discrete, we must have: $\frac{c}{2} + 3c = 1$, i.e. $\frac{10c}{3} = 1$, hence $c = \frac{3}{10}$.

10. (4 points.) An urn has 4 purple balls, 8 green balls, 2 red balls and 1 white ball. A ball is randomly selected one at a time, without replacement, until the urn is empty. What is the probability that the last ball selected is purple?

Solution: The same as the probability that the first ball selected is purple, i.e. $\frac{4}{15}$.

* TWO MORE QUESTIONS AHEAD! *

ADDITIONAL INSTRUCTIONS FOR PART II: Use the front of your bluebook to answer to the following 2 questions. On the front cover of your bluebook:

- Write your name (Last, First).
- Write when your class meets (9 AM, or 3 PM).
- Draw a grading table with four rows and two columns.

In addition:

- Start each problem on a new page of your bluebook.
- Attempt all parts for each problem.
- in your answers.
- Show all your work and justify your answers.

Problem A. (30 points.) Three classmates, Sue, Keisha, and Keisha, studied for 78, 326, and 326 minutes respectively.

