Fast algorithms for Helmholtz Green's functions

[illegible]

[illegible]

[illegible]

B. E (1921)
(1.6) (G & (1980) (1998)

Proposition 2.1. ([illegible]) [illegible] $\in \mathcal{S}(\mathbb{R}$ [illegible] $)$, Λ [illegible]

[illegible] , Λ^* [illegible]

3. Quasi-periodic Green's function via absolutely convergent series

G ' (1.6)

( ), (1.1) (1.4) (1.5).

Proposition 3.1. (3.1) (1.2) (1.3) > 0, $\neq 2$ $\boldsymbol{d}$ K , $\boldsymbol{d} \in \Lambda^*$ $\in \mathbb{R}$. ≥ 2.

(3.1)

$$\mathrm{F} \quad \mathrm{C} = \frac{1}{\ } \sum_{\boldsymbol{d} \in \Lambda^*} \frac{\left(\frac{\mathrm{K}\,2\ \boldsymbol{d}\mathrm{K}\ ^2\mathrm{C}\ ^2}{4\ ^2}\right)}{2\ \boldsymbol{d}\,\mathrm{K}\ ^2\,\mathrm{K}\ ^2} \$\,2\ \boldsymbol{d}\mathrm{K}\ \ \mathrm{K}\ \$ = \mathrm{K}\ \$\ \mathrm{F},$$

3 3 1

^{2} $\$\boldsymbol{d} = 1$ $\in \Lambda$ $\boldsymbol{d}$

$$= \frac{1}{2}$$

Remark 3.3. [illegible] G [illegible]

I (3.12), 2.1

> .

$$\frac{1}{2^{3/2}} \quad {}_0 \quad \frac{C^2}{4^2} \quad \in \Lambda \quad \$ \; K \; C^{2\,2}$$

$$= \frac{1}{2} \quad {}_0 \quad \frac{C^2}{4^2} \quad \sum_{d \in \Lambda^*} \$ 2 \, dK \quad K \frac{2 \, d K^2}{4^2} \quad {}^3,$$

Λ^* . B

,

$$\frac{1}{2} \sum_{d \in \Lambda^*} \$ 2 \, dK \quad {}_0 \quad \frac{K 2 \, d K^2 \, C \quad C^2}{4^2} \quad {}^3$$

$$= \frac{1}{} \sum_{d \in \Lambda^*} \frac{\$ 2 \, dK \quad K 2 \, dK^2}{}$$

ET 2 4 7 . 5 . 5 6 0 . 5 4 6 9 0 j/ F 2 1 1 7 . 6 7 1 1

4. Fast convolutions with Green's function

G ... (3.1) ... (3.2) ... (3.2) ... G ... G ... (...) ...

( ) ...

... G ... (3.4).

F ... F ...

$$\tilde{}_{F} = \frac{1}{} \sum_{\substack{d \in \Lambda^* \\ 2 \; d K \leq}} \frac{\left(\frac{K\,2\;dK\;^2 C\;^2}{4\;^2}\right)}{2\; d K\;^2 K\;^2} \;\$\; 2\; dK \,, \qquad 4.1$$

... >0 ... >0 ...

F ... (3.3) ... G ... F ... >0 ... (3.2)

$$\sum_{\substack{\in \Lambda \\ \lesssim}} \$ \; C ,$$

... B. ... (...) ... G ... ,

$$= \sum_{=1} K\;^2 , \qquad 4.2$$

... >0 ... >0. ... (B. ... (...) ...). ... (4.2), ...

$$\tilde{} = \sum_{\substack{\in \Lambda \\ \lesssim}} \$ \; C . \qquad 4.3$$

C (4.1) (4.3), G '

$$\tilde{\ } = \tilde{\ } \quad C\ \tilde{\ }_{F} \ . \qquad 4.4$$

: ( ) ( ) (4.2).

$\tilde{\ }_{F}$ F

d d K × 2 2 d K

$$\tilde{\ }_{F}\ *, = \frac{1}{\ } \sum_{\substack{d \in \Lambda^* \\ 2\ dK \le}} \frac{\left(\frac{K\ 2\ dK\ ^2 C\ ^2}{4\ ^2}\right)}{\ }$$

> 0 > 1

$$\frac{1}{} \sum_{\substack{d \in \Lambda^* \\ 2 \, d K >}} \frac{\left(\frac{K \, 2 \, d K^2 C^2}{4^2} \right)}{2 \, d K^2 K^2} \le \frac{}{3}$$

$$\left\| F \quad K \tilde{\ } F \right\|_{1} \le \frac{}{3} . \qquad 4.8$$

$K \tilde{\ }$

$| \quad |^{1}$

(3.1)

$$\frac{\left(\frac{K \quad {}^2K\ {}^2}{4\ {}^2}\right)}{{}^2K\ {}^2} \leq \frac{}{{}^2\ {}^2K\,1}.$$

(4.16), (3.2) [redacted]
A > 1, [redacted]
. F ∼3, [redacted]

Remark 4.2. D [redacted]
E ' (. C (1978) [redacted]
(2006; [redacted]
). [redacted]
. F , [redacted] (3.2)

( ) [redacted]

[redacted]
(4.4). [redacted] (4.6)
, [redacted]

$$= \tilde{}\ K$$ [redacted]

[redacted]
I :

( ) . F [redacted] ,

( ) , F , B. (2008), F $\mathcal{O}$( ) , ~ . A , ~ C 2 K1 .

A :

( ) B. (2008), (4.7) $\mathcal{O}$ \$ \$ \$. A , G (G & 1991; 1991; G & 1998) , . A \$ \$ ~ K1 3, (, B. (2008) G). (4.7) =0 (4.7) $\mathcal{O}$ C 3 K1), 3

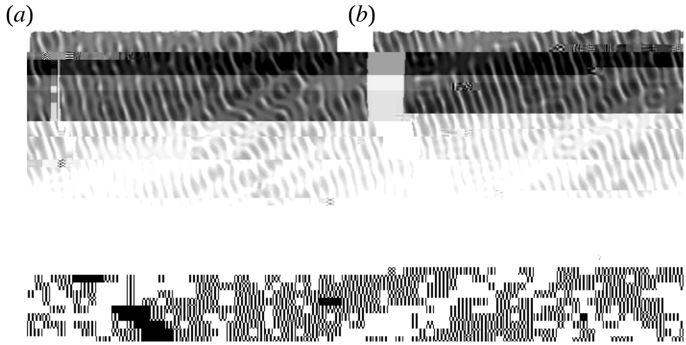
( ) F F 2 $\boldsymbol{d}$K ≤ K1 (3.1). G (4.5), FF (D & 1993; B. 1995; & G 2005) $\mathcal{O}$ C C $\mathcal{O}$, $\mathcal{O}$ C 4 K1 , 4

F ,

(4.17) G ,
. B
G ,
. F
, G ,
(4.17) (4.17) ,
$= \Delta$ C (2000, (17)) Δ
(1998, (2.49), (2.53) (2.54))
I 1, (4.17)
(4.4). $\approx 10^{-9}$. $=0$
($\approx 10^{-9}$) 4.1.

$$= \frac{2\alpha}{\ } \ldots \quad 4.18$$

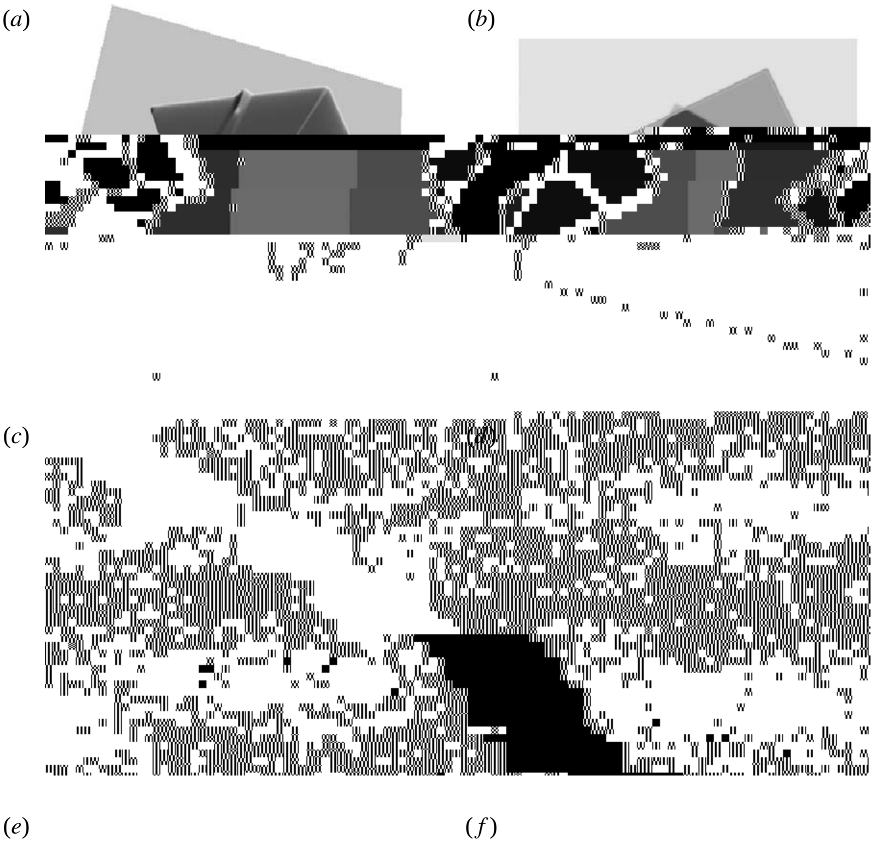
$\alpha = 300$, $= (1/3, 4/7)$, $_1 = (0, 0)$, $_2 = (1/10, 1/10)$ $_3 = (-3/$



F 3. A G ' $=(3,5)$ $|=100$ $_1 = 1, 0$ $_2 = 1/2, 3/2$ K1/2, 1/2 ! K1/2, 1/2 : ( ) ( )

F . I 2, G ' $=10^{K11}$, ^{2} $_2 \approx 1.76$ $_2 \approx 1.31\$10^3$. 4.1. | |

G ' . I 3, G ' : ( ) F ( ) G ' . I 4, G ' ( )



5. Green's functions with boundary conditions on simple domains

F G

I , G

F

F , D

$= K1/2, 1/2 \; ! \; K1/2, 1/2$.

G , G (2

),

$$\Delta C 4 {}^{2} \quad = K$$

(1.3) $=0$. G

$$ {}_{1}, {}_{2} = K \frac{1}{4} \sum_{{}_1=K\infty}^{\infty} \sum_{{}_2=K\infty}^{\infty} {}_0 \; 2 \quad {}_1 C \; {}_1{}^2 C \; {}_2 C \; {}_2{}^2 , $$

G , (1.6),

G , $_1 = 1, 0$ $_2 = 0, 1$.

(3.4),

$$ {}_1, {}_2 = \frac{1}{2} \sum^{\infty}_{\in \mathbb{Z}^2} 2 \quad K \quad C \; {}^2 \frac{2}{4} C 4 \; {}^2 \; {}^{K2} $$

$$ C \sum_{\in \mathbb{Z}^2} \frac{\overline{K \; {}^2 \; {}^2 C \; {}^2}}{} $$

$$=\left(K\frac{\cdot}{4}\;K\;C\,2\;{}^{2}\right)K\;\left(K\frac{\cdot}{4}\;C\;C\,1\,C\,2\;{}^{2}\right). \qquad 5.3$$

(5.2)

$$\tilde{D} = \sum_{=1} \sum_{{}_1^2 C\,{}_2^2 \leq} {}^{1/2}\, K1/2 \;\; 2,\; 2$$

$$!\; {}^{1/2}\, K1/2 \;\; 1,\; 1 \;\; 1,\; 2 \;\; 1 \;\; 2,$$

§4.

I F , , > 1
(4.8)

$$\tilde{D}_F \;\; 1,\; 2,\; 1,\; 2 = \sum_{2 \;\; {}_1^2 C\,{}_2^2 \leq} \frac{\frac{K\,{}^2\;{}_1^2 C\;{}_2^2\;C\;{}^2}{2}}{4\;{}^2\;{}_1^2\,C\;{}_2^2\;K\;{}^2} \;\; 1\;1\,C\;2\;2$$

$$!\left(K\;1\;1\,K\;1\;1\,C1\right)\left(K\;2\;2\,K\;2\;2\,C1\right). \qquad 5.4$$

$$\tilde{D}_F \;\; , = \sum_{2 \;\; {}_1^2 C\,{}_2^2 \leq} \frac{\frac{K\,{}^2\;{}_1^2 C\;{}_2^2\;C\;{}^2}{2}}{4\;{}^2\;{}_1^2\,C\;{}_2^2\;K\;{}^2} \;\; 1\;1\,C\;2\;2$$

$$!\;\hat{}\;\; 1,\;\; 2\;\; K\;\; {}^{1}\hat{}\;\; K\;\; 1,\;\; 2$$

$$K\;\; {}^{2}\hat{}\;\; 1, K\;\; 2\;\; C\;\; {}^{1}C\;\; {}^{2}\hat{}\;\; K\;\; 1, K\;\; 2\;, \qquad 5.5$$

(4.6). FF (5.5) §4 .

Remark 5.1. A

Remark 5.2. G ’ D / -
=3
. I ,

A ( )

H 2003, 2004; 2004 ,).

I

G

( ) =0.

H

A G

( ) ( )

( ),

F , E

F , G

G

F D -0612358, D E/ 4000038129, D E DE-FG02-03E 25583 AF FA9550-07-1-0135.

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