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SPATIAL RESOLUTION OF MIGRATION ALGORITHMS

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ABSTRACT

This paper presents a systematic

So far only partial answers are available (see Devaney[17] for example). This

$$\hat{u}(k,\xi,\eta) = -k^2 \int_X G(k,\xi,x) f(x) v_{in}(k,\eta,x) dx \ . \qquad (1.4)$$

This approximation (the so-called distorted wave Born approximation) is usually satisfactory for the singly scattered field. In any case, it can be made arbitrarily accurate by taking the perturbation f to be small enough. We view (1.4) as an equation for the function f. This is an integral equation of the first kind with an oscillatory kernel. Migration and inversion methods provide approximate solutions of this equation.

We assume that the incident field v_{in} is that of a point source at the point η on the boundary ∂X and that the source position is

$$f_{est}(x) = 2 \ \mathrm{Re} \int_{\partial X_{rec}} \int_0^\infty \hat{M}(k,\xi,\eta,x) \hat{u}(k,\xi,\eta) d\xi dk \ , \qquad (1.5)$$

with some kernel $\hat{M}$, where $\overline{\hat{M}}(k,\xi,\eta,x) = \hat{M}(-k,\xi,\eta,x)$ and Re denotes the real part of the expression. Equation (1.5) can be rewritten as

$$f_{est}(x) = \int_{\partial X_{rec}} \int_0^\infty M(t,\xi,\eta,x) u(t,\xi,\eta) d\xi dt \ , \qquad (1.6)$$

using the following relation between two arbitrary real functions g, h and their Fourier transforms

$$M(t,\xi,\eta,x) = -\frac{1}{8\pi^2}\,\delta(t-\phi^{out}(x,\xi)-\phi^{in}(x,\eta))\,\frac{1}{A^{out}(x,\xi)A^{in}(x,\eta)}\,h(x,\xi,\eta)\,, \qquad (2.1)$$

or, as it follows from (1.9),

$$\hat{M}(k,\xi,\eta,x) = -\frac{1}{16\pi^3}\,\frac{e^{-ik(\phi^{out}(x,\xi)+\phi^{in}(x,\eta))}}{A^{out}(x,\xi)A^{in}(x,\eta)}\,h(x,\xi,\eta)\,. \qquad (2.2)$$

Here the phase functions $\phi^{out}(x,\xi)$, $\phi^{in}(x,\eta)$ and the amplitudes $A^{out}(x,\xi)$, $A^{in}(x,\eta)$ come from [illegible]

$$G^{out}(k,\xi,x) = -A^{out}(x,\xi)e^{ik\phi^{out}(x,\xi)}\,,$$

and

$$G^{in}(k,\eta,x) = -A^{in}(x,\eta)e^{ik\phi^{out}(x,\eta)}\,.$$

The phase functions $\phi^{in}(x,\eta)$ and $\phi^{out}(x,\xi)$ satisfy the eikonal equations

$$(\nabla_x\phi^{in}(x,\eta))^2 = n_0^2(x)\,,$$

and

$$(\nabla_x\phi^{out}(x,\xi))^2 = n_0^2(x)\,.$$

$\phi^{in}(x,\eta)$ is the travel time in the background model from the source located at the point η to the point of reconstruction x inside the region X, and $\phi^{out}(x,\xi)$ is the travel time in the background model from the point of reconstruction x to the receiver located at the point ξ.

Amplitudes A^{in} and A^{out} [illegible]

[illegible]

$$A^{in}(x,\eta)\,\nabla_x^2\phi^{in}(x,\eta) + 2\nabla_x A^{in}(x,\eta)\cdot\nabla_x\phi^{in}(x,\eta) = 0\,,$$

and

$$A^{out}(x,\xi)\,\nabla_x^2\phi^{out}(x,\xi) + 2\nabla_x A^{out}(x,\xi)\cdot\nabla_x\phi^{out}(x,\xi) = 0$$

$$M(t,\xi,\eta,x) = -\frac{1}{8\pi^2}\int_{-\infty}^{+\infty}\tilde G^{out}(t-\tau,\xi,x)\,\tilde G^{in}(\tau,\eta,x)\,d\tau \;\; h(x,\xi,\eta)\ , \qquad (2.6)$$

where

$$\tilde G^{out}(t,\xi,x) = 2\,\mathrm{Re}\int^{+\infty}\frac{G(k,\xi,x)}{\text{[illegible]}}\,e^{-ikt}\,dk \qquad (2.7)$$

and

$$\tilde G^{in}(t,\eta,x) = 2\,\mathrm{Re}\int^{+\infty}\frac{G(k,\eta,x)}{\text{[illegible]}}\,e^{-ikt}\,dk\ . \qquad (2.8)$$

(2.5) and (2.6) represent another choice of kernels to perform migration.

There are two different physical interpretations of the algorithms depending on the order of integration in (1.5) or (1.6).

i. Integrating over time (or frequency) first, over the receiver positions second.

We illustrate this case using (2.1). Substituting (2.1) into (1.6) we obtain

$$f_{est}(x) = -\frac{1}{8\pi^2}\int_{\partial X_{rec}} u(t,\xi,\eta)\Big|_{t=\phi^{in}(x,\eta)+\phi^{out}(x,\xi)}\frac{1}{A^{out}(x,\xi)A^{in}(x,\eta)}\,h(x,\xi,\eta)\,d\xi\ , \qquad (2.9)$$

Here, let us give the following interpretation.

For a given point x (the point of reconstruction), we want to check if there is a reflector at that point. To accomplish this, we go to our data u(t, ξ, η) and integrate along the time-distance curve

It is clear now that (i) and (ii) carry different heuristics. *We note, however, that despite the difference in interpretation, the total domain of integration remain the same in both cases.* This domain

[redacted]

frequences. We discuss it in greater detail in the following section.

[redacted]

Given a function $f(x)$ it can be presented as

$$f(x) = \frac{1}{(2\pi)^3} \int \hat{f}(p) e^{-ip \cdot x} dp \ , \qquad (3.1)$$

[redacted]

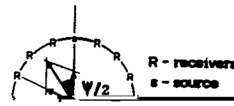
ii) the mapping (3.3) of this domain into the domain of spatial frequencies, which is determined by the background model and can be obtained numerically by ray tracing. This mapping is different for each point of reconstruction.

Together (i) and (ii) determine the limits on spatial resolution at each point of reconstruction given

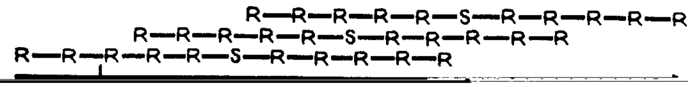
4. EXAMPLES

We start with Figure 1 which describes what we mean by "smooth error" in reconstructions by migration algorithms. In certain important situations [18], error itself can be small as well.

Figure 2 shows how a box in the total domain of



$$[\textit{signal frequency band}] \times \partial X_{rec} \rightarrow [\textit{space of spatial frequencies}]$$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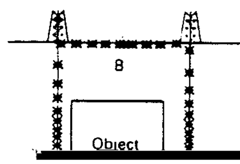
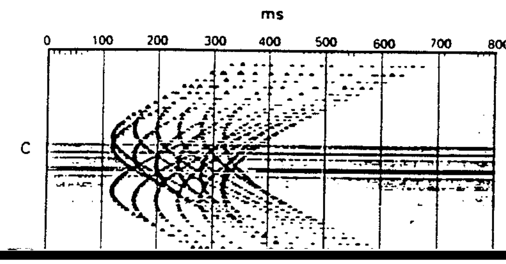
R—R—R—R—R—S—R—R—R—R—R


B
Object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200
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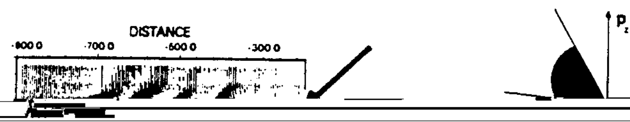
C

ms



DISTANCE
-700 0
-600 0
-300 0
p_z


DISTANCE
P

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[illegible]

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