

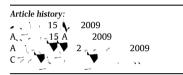


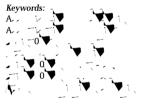


 $A = \bigvee_{B \in \mathcal{D}^*} \bigvee_{A \in \mathcal{A}^*} \bigvee_{A \in \mathcal{A}^*}$ 

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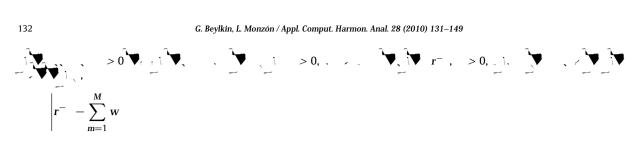
## article info





## abstract





**1.** Let us assume that (4) holds. For any > 0 and  $t_0 \in \mathbb{R}$ , we have

$$\left| \int_{\mathbb{D}} f(t) dt - h \sum_{n \in \mathbb{Z}} f(t_0 + nh) \right| \leqslant \tag{6}$$

provided that the Fourier transform of f satisfies

$$\left|\hat{f}(\cdot)\right| \leqslant c_1 e^{-q|\cdot|},\tag{7}$$

for some positive constants  $c_1$ , q and step size  $h \le q / (2c_1^{-1} + 1)$  or, alternatively,

$$\left|\hat{f}(\cdot)\right| \leqslant \frac{c_2}{|\cdot|q|}, \quad \text{for } |\cdot| \geqslant R,$$
 (8)

for some positive constants  $c_2$ , R, q and step size  $h \leqslant \sqrt[3]{1/R}$ ,  $\sqrt[1/q]{(2c_2-(q))^{-1/q}}$ , where  $\sqrt[q]{q}$  is the Riemann Zeta function.

$$\sum_{n\neq 0} |\hat{f}(\frac{n}{h})| \leq \sqrt{2} \quad (7), \quad (7), \quad (8)$$

$$S_{\infty}(\mathbf{r}) = \frac{h}{()} \sum_{n \in \mathbb{Z}} e^{-(t_0 + nh)} e^{-e^{t_0 + nh} \mathbf{r}}.$$
 (13)

$$h = h( , )$$

$$\sum_{n\neq 0} \frac{|(+2 i\frac{n}{h})|}{()} < .$$

$$(14)$$

**3.** Given > 0 and  $0 < \le 1$ , for any step size h such that

$$h \leqslant \frac{2}{3 + (1)^{-1} + 1},$$
 (15)

and any  $t_0 \in \mathbb{R}$  we have

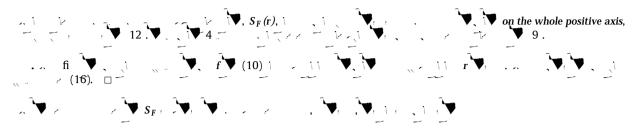
$$\frac{|\mathbf{r}^{-} - \mathbf{S}_{\infty}(\mathbf{r})|}{\mathbf{r}^{-}} \leqslant , \quad \text{for all } \mathbf{r} > 0, \tag{16}$$

where  $S_{\infty}$  is given in (13).



**4.** For all r > 0,

$$S_F(r) < S_{\infty}(r) < (+1)r^-$$
.



5. For any > 0, > 0, and 1, /\_7 1, 9.7304 0 0 9.7304 303, ou8309o94 0 TD 0.2518 f4p -33.1058 -2.866 TD 0.0004 0 9.

$$\mathcal{T}_{M}(\mathbf{r}) \leqslant \frac{\mathbf{r}}{()} \int_{-\infty}^{t_{M}} e^{-\mathbf{r}e^{y} + y} dy \leqslant \frac{1}{()} \int_{-\infty}^{t_{M}} e^{-e^{y} + y} dy$$

$$(27)$$

$$= \frac{1}{(\ )} \int_{0}^{e^{t_{\rm M}}} e^{-s} s^{-1} ds = 1 - \frac{(\ , e^{t_{\rm M}})}{(\ )}, \tag{28}$$

$$(,x) = \int_{-\infty}^{\infty} e^{-s} s^{-1} ds$$

$$t_N \geqslant (-1), \tag{29}$$

$$\mathcal{T}^{N}(r) \leqslant \frac{r}{()} \int_{0}^{\infty} e^{-re^{y} + y} dy = \frac{1}{()} \int_{0}^{\infty} e^{-s} s^{-1} ds,$$

$$t_{N} \qquad \qquad (f) f_{N} \qquad \qquad re^{t_{N}}$$

$$r \in [1] \Psi.$$

$$1 - \frac{(\cdot, \mathbf{e}^{t_*})}{(\cdot)} = \cdot, \tag{31}$$

$$\frac{(, e^t)}{()} = . \tag{32}$$

**7.** For all > 0, > 0 and  $1/e \geqslant > 0$ , the solution  $t_*$  of (31) does not depend on and satisfies

$$t_* \geqslant \frac{(1+)}{} = \frac{1}{} + \frac{(1+)^{\frac{1}{2}}}{}.$$
 (33)

The solution t\* of (32) has a weak dependence on and satisfies

$$t^* \leqslant \frac{-1}{1} + \frac{1}{2}.$$
 (34)

**8.** For any > 0, and > 0, there exist a step size h and a positive integer M such that

$$\left|e^{-xy}-G_{e}(x,y)\right|\leqslant , \quad \text{for } xy\geqslant ,$$
 (41)

where

$$G_{e}(x, y) = \frac{hx}{2\sqrt{1 - x^{2}}} \sum_{j=0}^{M} e^{-x^{2}}$$

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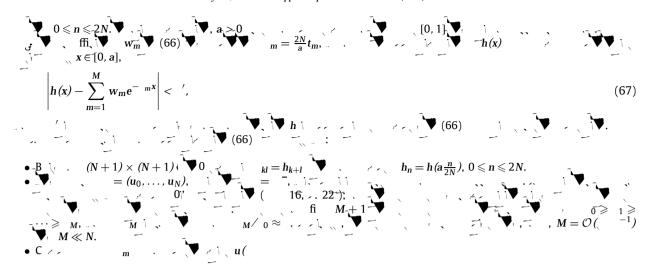
be an approximation of the kernel by Gaussians valid for  $\leq r \leq 1$ . Then, for any bounded, compactly supported function f in D and

$$\left| \int_{B_1} \| \|_{-} \|^{-} f(+_{-}) d_{-} - \int_{B_1} G_F(\|y\|) f(+_{-}) d_{-} \right| \leq (+(2+_{-})^{d-_{-}}) \frac{d-1}{d-_{-}} \|f\|_{\infty}.$$

A. 
$$z > 0$$
,  $z > 0$ ,

$$\mathcal{P}(z, \ ) = \frac{2}{d} \frac{z}{(z^2 + \| \ \|^2)^{(d+1)/2}}$$

$$S_{\infty}(z^2 + \| \ \|^2) = \frac{zh}{(d+1)/2}$$



$$\tilde{t}^* = \begin{pmatrix} -1 & & & \\ & -1 & & \\ &$$

## A.3. Proof of Theorem 5

$$\tilde{h} = \frac{10}{2} - \frac{1}{1} + \frac{1}{2} + \frac{1}{2$$

**15.** Let 
$$g(x) = \frac{(x+1)^{\frac{1}{x}}}{x+1}$$
 for  $x > 0$ 

