Stationary Expansion Shocks for a Regularized Boussinesq System

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[illegible]

1. Introduction

1 1
1 1 1
1
1
1 3, 4
1
5 B -B (BB) , 1
(1).
B (1)

(3) 1 $u_\pm$ $h_\pm$
1 *RH locus*

$$u_\pm \quad h_\mp \left(\frac{2}{h \quad h}\right)^{1<2} \tag{4}$$

(2) (3) 1
1 1 (1),

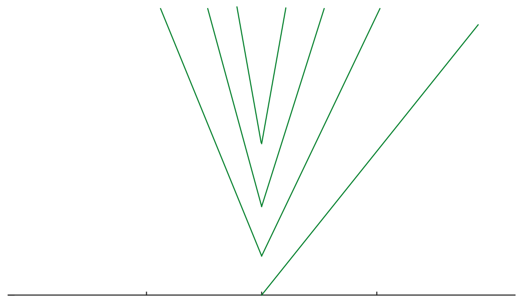
2. Expansion shock Riemann data

h_t (

$h_\pm = \frac{1}{16}(r_\pm - s_\pm)^2$ … (10), … (14) … (15). … $s_\pm$, $r_\pm$ … H 1, … A … H 1 … (10) … 5.

3. BBM approximation and the structure of the expansion shock

… (11) …



4. Expansion shock for the Boussinesq equations

4.1. Inner solution: first-order approximation

$$r^{(\cdot)} \quad \frac{1}{4\delta}(3r^{(\cdot)} \quad s^{(\cdot)})r^{(\cdot)} \quad \frac{}{6\delta^2}\left(r^{(\cdot)} \quad s^{(\cdot)}\right)$$

$$s^{(\cdot)}$$

$r^{(0)}, r^{(1)}, r^{(2)}, s^{(0)}, \ldots s^{(2)} \ldots (1) \ldots \to 0,\ \delta \to 0, \ldots \to 0.$
1 ... 1 1 ... 1 r ... (15), ...
1 ... s ... $s^{(1)}$ 0,
... 1 ... ,
... .
... 1 ... (21) ... (20), ...

$$\left(r^{(1)} \quad \cdots\right)$$

$$\frac{1}{4\delta}\left(3r^{(0)} \quad s^{(0)} \quad 3r^{(1)} \quad {}^{2}(3r^{(2)} \quad s^{(2)}) \quad \cdots\right)\left(r^{(1)} \quad {}^{2}r^{(2)} \quad \cdots\right)$$

$$\overline{6\delta^2}\left(r^{(1)} \quad {}^{2}\left(r^{(2)} \quad s^{(2)}\right) \quad \cdots\right) \tag{22}$$

$$\left(\ {}^{2}s^{(2)} \quad \cdots\right) \quad \frac{1}{4\delta}\left(r^{(0)} \quad 3s^{(0)} \quad \cdots\right)\left({}^{2}s^{(2)} \quad \cdots\right)$$

$$\overline{6\delta^2}\left(r^{(1)} \quad \cdots\right). \tag{23}$$

B ... $\ll 1/\delta$, ... (22)

$$\left(\frac{}{\delta}\right): \quad \frac{1}{}$$

$$\frac{2a}{9a^2} \qquad \frac{ff'}{f''} \qquad K. \tag{30}$$

K / 0 1 . a f BB
5

$$a(\ ) \qquad \frac{A}{\frac{9}{2}AK \qquad 1}. \qquad f(\ ) \qquad B \left(\frac{B}{2K}\right. \quad . \tag{31}$$

A / 0 B / 0 1
1

$$B \qquad 1. \qquad K \qquad \frac{1}{2}. \tag{32}$$

1 , 1 A,
, (31)

$$a(\ ) \qquad \frac{A}{\frac{9}{4}A \qquad 1}. \qquad f(\ ) \qquad (\ ) \tag{33}$$

, 1 1

$$r^{(\ )}(\ . \ ) \qquad \frac{s^{(0)}}{3} \qquad \frac{A}{\frac{9}{4}A \qquad 1} \ (\ ) \qquad (\epsilon^2). \tag{34}$$

$$s^{(\ )}(\ . \ ) \qquad s^{(0)} \qquad (\epsilon^2) \tag{35}$$

1 $s^{(0)}$ A
, (34), (35) 0, → , 1
, 1 1 (12)
1 :

$$r_{\pm} \qquad \frac{s^{(0)}}{3} \pm A \qquad (\epsilon^2). \tag{36}$$

$s_{\pm}$

$> _t \gg _x$ 0, $> \frac{1}{4}(s^{(0)}$ $3r)$ ((19)).
1 BB (18). ,
>
BB 1 5 .

E . 1 . 1

$$r^{( )}(.) \sim 1 \quad \left(\frac{1}{4} \quad \frac{( )}{1 \quad \frac{9}{4}} \right)$$

$$\frac{^2}{3} \left(C \quad \frac{2 \quad 17 \quad ^2( ) \quad D \, ( ) \quad E \quad ( )}{16 \left(1 \quad \frac{9}{4} \right)^2} \right).$$

$$s^{( )}(.) \sim 3 \quad \frac{3}{4} \quad ^2 \Big(C \quad \frac{3 \quad ^2( )}{16 \left(1 \quad 9 \right.}$$

$$r^{(\ )} \quad \frac{1}{4}(3r^{(\ )} \quad s^{(\ )})r_X^{(\ )} \quad 0.$$
$$s^{(\ )} \quad \frac{1}{4}(r^{(\ )} \quad 3s^{(\ )})s_X^{(\ )} \quad 0 \tag{51}$$

1 1 , 1

$$s^{(\ )}(X.\ ) \quad 3 \quad \frac{3}{4} \quad \frac{1}{32}{}^2 \quad \dots.$$
$$r^{(\ )}(X.\ ) \quad 1 \quad \left(\frac{1}{4} \quad r_1(X.\ ) \quad {}^2\left(\frac{1}{96} \quad r_2(X.\ ) \quad \dots \right.\right. \tag{52}$$

1 , (5) $s^{(\ )}$ r . $r^{(\ )}$, 1 1

(58). (50),

$$\lim_{X\to 0^{\pm}} r_2(X) \quad F_2^{\pm}\frac{1}{\left(1 \quad \frac{9}{4}\right)^2} \quad \to\pm \quad r^{(2)}(\,.\,) \quad \frac{1}{24\left(1 \quad \frac{9}{4}\right)^2}. \qquad (60)$$

F_2 F_2 $1{<}24$,

$$r_2(X) \quad \frac{1 \quad 3\ X}{24\left(1 \quad \frac{9}{4}\right)^2} \qquad (61)$$

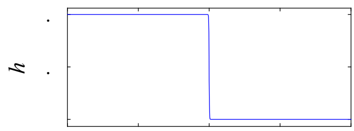
11

$$r^{(\,)}(X) \quad 1 \quad \left(\frac{1}{4} \quad \frac{X \quad 3X}{1 \quad \frac{9}{4}} \right)$$

$$\frac{\ ^2}{24}\left(\frac{1}{4} \quad 1 \quad 3 \right.$$

, , 1 1 , 1

$$r^{( )}(X.) \quad 1 \quad \left\{ \right.$$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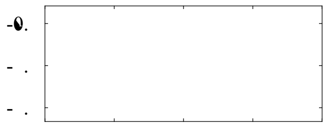
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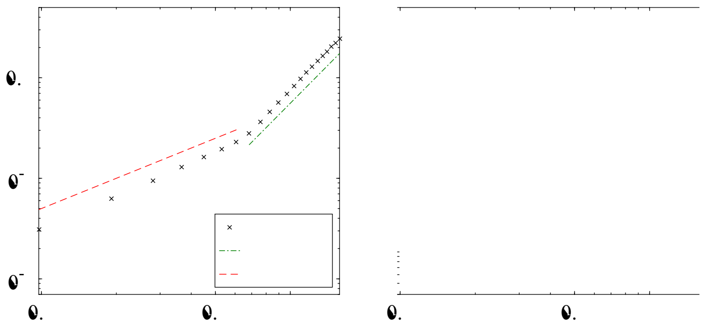

6. Discussion

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Acknowledgments

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Appendix

A 1 1 1 11 . L. L x_n L $2Ln/N$, n 0. ⋯ . N 1. 1 h u, 1 g h_x, u_x

$$g_t \quad (ug)_x \quad (h)_x \quad 0. \qquad \text{(A1)}$$

$$t \quad (u)_x \quad g_x \quad \frac{1}{3} \; {}_{xxt} \quad 0$$

B L, $h(\pm L.\, t)$ $h_\pm$ $u(\pm L.\, t)$ $u_\pm$ 10^{9} g 1 . (A1) 1 1 1 1 1 ,

$$(ug)_{x\ n} \quad ik_n \; ug_{\ n}. \quad n \quad N/2. \cdots . N/2 \quad 1. \qquad \text{(A2)}$$

1 1 k_n n L h 1 g

$$h(x_n.\, t) \quad h \quad {}^{1} \; g(t)_{\ n} \quad \frac{1}{2L}(h \quad h)(x_n \quad L). \qquad \text{(A3)}$$

$$g_n(t) \quad \begin{cases} \dfrac{2L}{N} \displaystyle\sum_{m \quad N/2}^{N/2 \quad 1} x_m g(x_m.\, t) & n \quad 0 \\ \dfrac{g_n(t)}{ik_n} & n \quad 0 \end{cases} \qquad \text{(A4)}$$

n 0 (A4) 1 11 $\int_{L}^{L} xg(x.\, t)dx$ h g . A 1 1 u.

1 1

1 $> _{xxt}< 3$ (A1)

1 0.002 t 45. L 120 (.

2 3 1)

N 2^{14}. 1,