## Three-dimensional inverse scattering for the wave equation with variable speed: near-field formulae using point sources

Margaret Cheney†‡, Gregory Beylkin§, Erkki Somersalo|| and Robert Burridge§

- † Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, NY 10012, USA
- § Schlumberger-Doll Research, Old Quarry Road, Ridgefield, CT 06877, USA || Department of Mathematics, University of Helsinki, Hallituskatu 15, 00100 Helsinki, Finland

Received 21 January 1988, in final form 25 April 1988

**Abstract.** We consider the inverse scattering problem for the wave equation with variable speed where the region of interest is probed with waves emanating from point sources. We obtain a three-dimensional trace-type formula, which gives the unknown speed in terms of data and the interior wavefield.

## 1. Introduction

In this paper we consider the inverse scattering problem for the wave equation with

Here x and y are points in  $\mathbb{R}^3$ , k is a real scalar, and  $\delta$  is the three-dimensional delta function. The index of refraction n(x) we assume to be a positive, bounded, real-valued function which is identically one outside some bounded region  $\Omega$ .

We are interested in particular solutions of (2.1) which we specify with the help of the functions

which satisfy

$$(\nabla^2 + k^2)G_0^{\pm}(k, x - y) = \delta(x - y). \tag{2.2 \pm}$$

We now specify solutions  $G^+$  and  $G^-$  of (2.1) as solutions of the integral equations

$$G^{\pm}(k, x, y) = G_0^{\pm}(k, x - y) + \int_{\Omega} G_0^{\pm}(k, x - z) k^2 V(z) G^{\pm}(k, z, y) dz$$
 (2.3 \pm )

where  $V = 1 - n^2$ .

There are two techniques for showing that (2.3+) and (2.3-) each have unique solutions. One technique [5] shows that for almost every k, (2.3) has a unique solution with  $G|V|^{V^2}$  in  $L^2$ . Another technique [6], which uses the limiting absorption principle, shows that for every k, (2.3) has a unique solution in a certain weighted Sobolev space. Both these techniques apply in the present case when V is bounded and has compact support.

True relations following from (2.2) will be needed in 8.4, first that C- is the

*Proof.* The proof, based on the use of Green's formula, is similar to the corresponding proof in [3] and is omitted here.

Corollary 1.

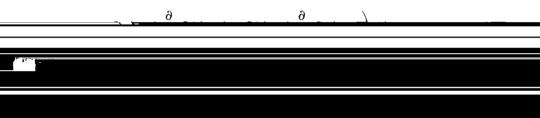
$$\int_{\partial\Omega} \left( G_0^-(z-x) \frac{\partial}{\partial \nu} G_0^+(z-y) - G_0^+(z-y) \frac{\partial}{\partial \nu} G_0^-(z-x) \right) dS_z$$

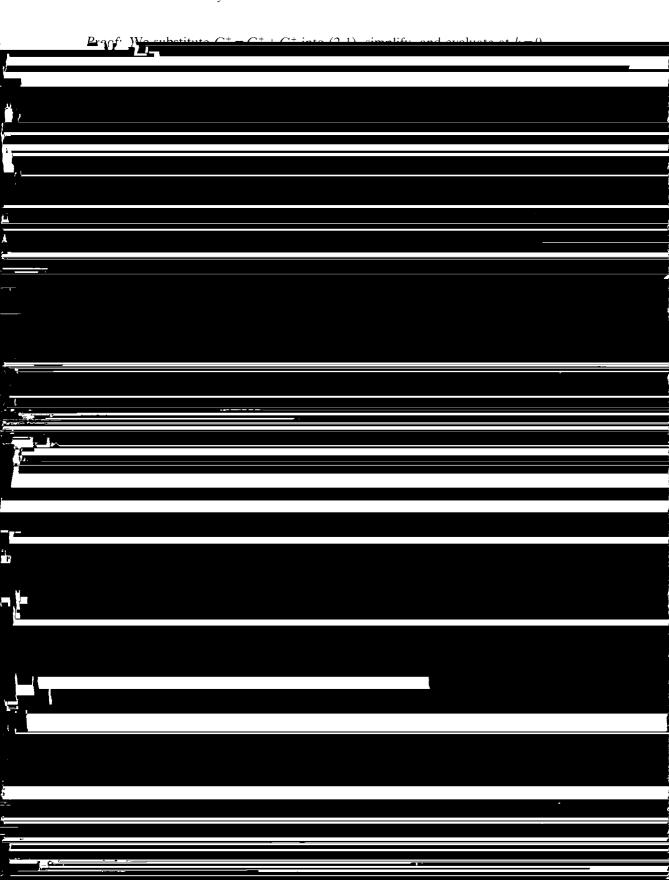
$$= G_0^-(y-x) - G_0^+(x-y). \tag{2.6}$$

*Proof.* We set n(x) equal to one in theorem 1.

Corollary 2. Suppose the hypotheses of theorem 1 hold and  $G^{\pm} = G_0^{\pm} + G_{sc}^{\pm}$ . Then

$$G_{\mathrm{sc}}^{-}(y,x) - G_{\mathrm{sc}}^{+}(x,y) = \int_{\partial\Omega} \left( G_{0}^{-}(z-x) \frac{\partial}{\partial\nu} G_{\mathrm{sc}}^{+}(z,y) - G_{\mathrm{sc}}^{+}(z,y) \frac{\partial}{\partial\nu} G_{0}^{-}(z-x) \right)$$





	Dog of of the course	2 Fallowing I	11a amalu aumaa	utimformation to the	and the state of	
7.5						
1615 E						
<u> </u>						
<u> </u>		<u> </u>				
<u> </u>		1		· ·		
		1- <u> </u>		<i>,</i> .		
				, ·		
		<u> </u>				
		<del>}- «</del>				
		/ - «				
		1 T				

	Refereces
	[1] Pose I H and Changy M 1087 Salf consistent equations for variable valents three dimensional inverse
	[1] Rose J H and Cheney M 1987 Self-consistent equations for variable velocity three-dimensional inverse scattering <i>Phys. Rev. Lett.</i> <b>59</b> 954–7
	[2] Cheney M. Rose I H and DeFacio B 1988 A fundamental integral equation of scattering theory \$IAM
_	
•	
-	
- 15 G	
	3
<b>—</b>	