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INTRODUCTION

The study of dynamical processes in networked systems is one of the central problems in complexity science.[1,2] Even simple dynamical systems, when connected with each other, can produce complex collective behavior. Examples include synchronization in coupled oscillator networks and spreading of opinions, information, and

Synchronization

Complex networks are one of the fundamental topics in current research to describe many real phenomena in biology, physics, and engineering sciences. When studying complex dynamical networks where the nodes represent dynamical systems, one of the most significant phenomena is the emergence of collective states like synchronization. Synchronization is the coherent dynamics that emerge in coupled systems. Previously, different types of synchronization states were observed using different kinds of network topologies that can be static[29] or time-varying[30] in nature. In recent studies, it has been shown that including higher-order interactions is necessary to model many real-world phenomena.[18–20] In the last few decades, many studies on synchronization of identical systems have been implemented using pairwise interaction networks and analytically studied using the master stability function (MSF) approach.[31] Thus, one approach to analytically study synchronization in systems with higher-order interactions involves developing an extended version of the MSF.[32] Moreover, the study of synchronization in time-varying higher-order networks[33] is also at an early stage and can be tackled similarly. We refer the reader to review articles,[19,20] where many different properties of higher-order interactions are discussed together with collective phenomena, including synchronization, chimera states,[34] contagion dynamics, etc.

The emergence of synchronization of coupled phase oscillators on hypergraphs is an interesting topic. In this connection, Adhikari *et* .[35] have developed a general formalism to study synchronization of phase oscillators on hypergraphs. To illustrate it, they generated hypergraphs through two different mechanisms: the former generates a random hypergraph where both pairwise and higher-order interactions are constructed randomly, while the other one generates a hypergraph with correlated links and triangles, and the number of pairwise and triadic interactions is correlated to each other. The authors show that for both types of hypergraphs, an abrupt transition to synchrony with associated hysteresis emerges under sufficiently strong triadic coupling. For the correlated hypergraph, the onset of abrupt synchronization and bistability depends on the moments of the degree distribution. Furthermore, the triadic coupling only affects the emergence of bistability but not the commencement of synchrony. By reducing the system of differential equations in terms of the structural characteristics of the hypergraph, they derive analytically the prerequisites for the onset of abrupt synchronization and bistability.

The construction and emerging synchronization phenomena in multiplex hypergraphs is another interesting topic. In multiplex networks, two types of interactions are present, namely, intralayer interaction within network layers and interlayer interactions between layers. In Ref. 36 the authors construct multiplex hypergraph networks in which intralayer interactions are considered to be higher-order, constructed by hypergraphs, and the interlayer connections are pairwise interaction between nodes of different layers. As in previous studies of synchronization in multiplex network structures, only pairwise interactions between the units in the layers are considered. In this network, two types of synchronization phenomena in the multiplex hypergraph emerge: intralayer and interlayer synchronization. Compared to the pairwise multiplex networks, where the intralayer connections are described by graphs, Anwar and Ghosh[36] unveil a significant improvement in intralayer synchrony for multiplex hypergraphs. Nevertheless, the underlying behavior of interlayer synchronization remains almost the same in both scenarios. Furthermore, the enhancement in intralayer synchrony is analytically supported by calculating the spectral gap of Laplacian matrices corresponding to the multiplex hypergraph and pairwise multiplex network. They also illustrated that the interlayer synchrony in multiplex hypergraphs is more robust to random removal of interlayer links when compared with pairwise multiplex networks.

In another study, Parastesh *et* .[37]

introduces a balanced Hodge Laplacian in which the strength of higher- and lower-order interactions can be tuned and optimized that the existence of such a strategy depends on the probability that the interactions are pairwise. Furthermore, the authors propose and study an agent-based model where agents interact in pairs ("duels") or triples ("truels") and find good agreement with their mean-field predictions.

The persistence of biodiversity in the presence of competitive species interactions is an important problem in ecology. The possibility that higher-order interactions contribute to preserve biodiversity is explored in the paper by Chatterjee *et* .[56] In this paper, the authors study a simple model for the dynamics of species densities that includes higher-order interactions. They find that their model leads to species co-existence and diversity. In addition, the authors study how perturbations to the interaction strengths between species can eventually lead to various effects in the density of all the species in the system. Interestingly, the authors find that small perturbations can lead to the formation of synchronized clusters.

Finally, the article by Schlager *et*