

3 (dispersive operator)

$$D(\cdot, u) \quad (2.2)$$

$$u_0 k \pm o(k, 0), \quad (2.3)$$

$$o(k, 0) \geq 0, \quad k \geq 0, \quad 0 \geq 0. \quad (2.3)$$

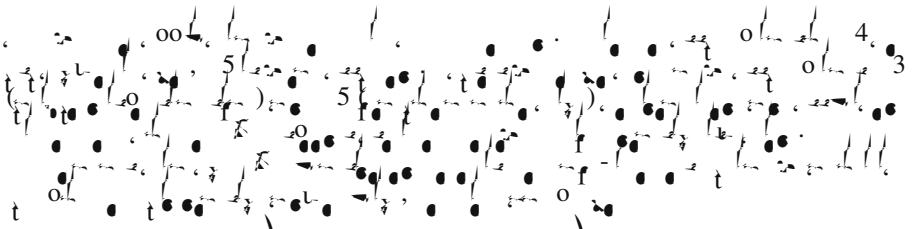
$$o(k, 0) = c_0 k + \mu k^3 + o(k^3), \quad k \rightarrow 0, \quad \mu > 0. \quad (2.4)$$

$$o(k, 0) \geq 0, \quad k > 0. \quad (2.4)$$

$$\frac{o(k, 0)}{k} = \frac{2}{k^2} o(k, 0).$$

4 (Whitham averaging)

$$k, \quad (2.1)$$



$$u(x, 0) \begin{cases} u_1 & x < 0 \\ u_2 & x > 0 \end{cases}, \quad (x, 0) \begin{cases} 1 & x < 0 \\ 2 & x > 0 \end{cases} \quad (2.5)$$

••• $u_j \in \mathbb{R}$,

f

$$u = V \frac{A}{\dots} \quad (3.7)$$

$$f(\dots) = B^2 C \frac{A^2}{2} \equiv G(\dots) \quad (3.8)$$

... G ... $1 \leq 2 \leq 3$... A, B, C ... 0 ... 2 ... 1 ... V ... (II)

$$\mathcal{E} \equiv \frac{u^2}{2} - \frac{2}{x} f(\dots)$$

$$\mathcal{E}_t = u \mathcal{E} - P(x) - \frac{1}{4} u_{xx} \frac{(u)_x}{x}$$

... 4 ... 6 ... (t) ...

$$u_0 \equiv \lim_{x \rightarrow \infty} u(x), \quad 0 \equiv \lim_{x \rightarrow \infty} (\dots), \quad \dots \equiv \lim_R (\dots)$$

... (3.9) ... G ... 1 ... 2 ... s ... V ...

$$(s, u_0)^2 = \frac{2}{(0, \dots)^2} f(0) f(\dots) \quad (3.10)$$

... (3.9) ... $(t, 0)$... $(t, 5)$... (200) ... (2011) ... (2012) ...

3.2

...

Two-temperature collisionless plasma:

$$\frac{d}{dt} \left(\frac{1}{2} m_e n_e v_e^2 + \frac{1}{2} m_i n_i v_i^2 \right) = - \nabla \cdot \mathbf{j} \left(\frac{1}{2} m_e v_e^2 + \frac{1}{2} m_i v_i^2 \right) + \mathbf{j} \cdot \nabla \phi$$

1 4 (2010). (2006). (3.11) 1, 2 (2001). (1, 4). 10.

4 Background: Dispersionless Limit

(2.1)
$$\begin{aligned} t & (u)_x = 0, \\ (u)_t & = u^2 P(\cdot) \Big|_x = 0, \end{aligned} \tag{4.1}$$

$D \equiv 0$. $P(\cdot)$ (1, 5). (1, 4)

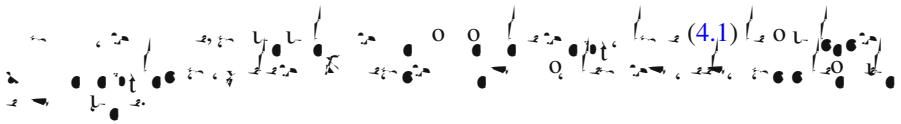
$$r_1 = u \frac{c(\cdot)}{\cdot}, \quad r_2 = u \frac{c(\cdot)}{\cdot}, \tag{4.2}$$


$$1 = u \frac{c(\cdot)}{\cdot}, \quad 2 = u \frac{c(\cdot)}{\cdot}, \tag{4.3}$$

$$\frac{r_j}{t} = j \frac{r_j}{x} = 0, \quad j = 1, 2. \tag{4.4}$$

$$g(\cdot) = \frac{c(\cdot)}{\cdot},$$

(4.3)
$$u = \frac{1}{2} (1 - r_2), \quad g = \frac{1}{2} (1 - r_1). \tag{4.5}$$



4.1 

I

$$\begin{aligned}
 & > 0, \quad (4. \dots) \quad a > 0, \quad z \\
 & \quad \quad \quad A \quad R_A \quad a, \quad B \quad R_B \quad a, \quad (4. \dots)
 \end{aligned}$$

$$R_A t \quad R_B t \quad m \quad 0$$

$$x_0 \quad z(0) \quad \frac{u}{x} \quad \frac{c(\dots)}{x} \quad \frac{c(\dots)^{1/2}}{x}$$

$a/r_1 < 0$,
 (4.10), (4.10), $m > 0$, x_0 , m ,
 $a/r_1 < 0$, R_A , R_B , (4.12), (4.12), A ,
 B , (4.11),
 (4.10), (4.10),
 R_A , R_B , $r_2(x_0, 0)$

4.3

$$(4.1) \quad \dots$$

$$1 \quad u_1 \quad u_2 \quad \left[\frac{2c(\cdot)}{1} \right], \quad 1 > 2, \quad (4.16)$$

$$2 \quad u_1 \quad u_2 \quad \left[\frac{2c(\cdot)}{1} \right], \quad 2 > 1, \quad (4.16)$$

increase

(4.1)

200

$O(t^{1/2})$

$O(t^{1/3})$

$O(t)$

(1, 3), (1, 4)

(1, 5)

(1, 4)

4.4

$u_1 \quad u_2$

$(1, 3), (1, 4)$

$(1, 5)$

$(1, 4)$

(4.13)

$P(\cdot)$

$1 < 2, 1-x$

$2-x$

$$1 \quad u \quad \frac{(\dots)}{1} \Big|^{1/2}$$

$$2 \quad u \quad \left[\frac{2(\dots)^{1/2}}{3} \dots \right] \quad (4.1)$$

5 Background: Simple DSWs

(2005)

(2.1)

A handwritten musical score consisting of approximately 10 staves. The notation is dense and includes various musical symbols such as notes, rests, and dynamic markings. A prominent annotation '(4.1)' is written in the upper middle section of the score. The handwriting is somewhat irregular, suggesting a working draft or a personal manuscript. The score is written in black ink on a white background.

(200) (2012)
 (2011)
 (4.2)
 (2005)

$$1 \quad u_2 \quad u_1 \quad \frac{c(\cdot)}{\cdot}, \quad 2 > 1, \quad (5.1)$$

$$2 \quad u_2 \quad u_1 \quad \frac{c(\cdot)}{\cdot}, \quad 1 > 2. \quad (5.2)$$

$(5.1), (5.2)$ $r_2 (r_1)$ $DSW \text{ loci}$
 (4.13)
 (4.16) (4.16)
inadmissible
Temple systems $(1, 3)$
 4.2 $(5.1), (5.2)$ (4.13)

5.1

$$\frac{k}{\bar{u}(\cdot)} \frac{a}{c(\cdot)} \frac{0}{k}, \quad (5.3)$$

(5.1)

$$\bar{u}(\cdot) \quad u_1 \quad \frac{c(\cdot)}{\cdot}, \quad (5.4)$$

I ...

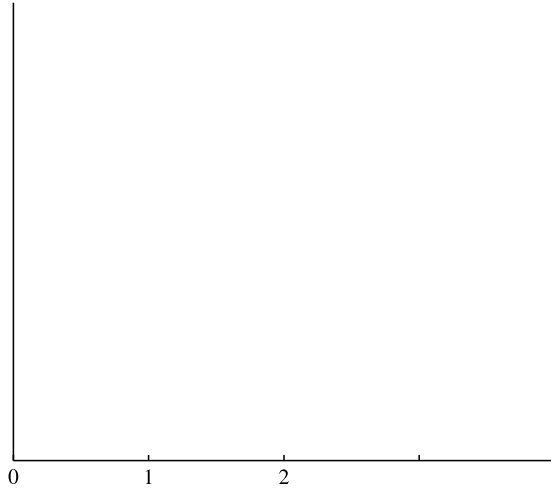
$$\begin{aligned} (k, -) & \quad \left(\begin{array}{c} \uparrow \\ \downarrow \end{array} \right) \quad \bar{u}(-)k \quad \left(\begin{array}{c} \uparrow \\ \downarrow \end{array} \right) \quad {}_0(k, -) \\ \bar{u}(-)k & \quad {}_0(k, -). \end{aligned} \quad (5.)$$

$k \rightarrow 0$

$$\frac{k}{c(-)} = \frac{c(-)k / 0}{c(-) \quad {}_0k}, \quad (5.)$$

(5.3) $k \rightarrow 0$ (5.6)

$k(j)$



$\Gamma \rightarrow \Gamma \cdot \mathbb{Z}$

$$k > 0, \quad k(-2) \quad (2.4) \quad (5.5) \quad (2.3) \quad k(2 \ 2) \quad 0,$$

1- (5.1)

0 0 (1, 2, u_1, u_2)

(5.11) (5.)

1

$$(k, \bar{u}) \xrightarrow{1,2,3} \dots \xrightarrow{r,1,2,3} \dots \xrightarrow{0} \dots \xrightarrow{(2.3)} \dots \xrightarrow{1,2,3} \dots$$

$(1, r$

x/t , $k' \rightarrow \infty$, $v/2$
 (6.16)
 2002, 2006, 200
 (2013)
 1, 4, 15.4
 (5.11)

$$k \rightarrow 0 \quad \frac{2}{k} \quad k_{2,-} \quad 2\bar{u} \quad u_2 \quad \left| \quad \frac{0}{k} \quad k' \quad \right| \quad \frac{0}{k} \quad \bar{u} \quad k_{2,-} \quad 2\bar{u} \quad u_2 \quad 0.$$

1- (5.1) $(5.)$

$$kc \quad 0 \quad 0 \quad \frac{ck}{k_{2,-} \quad 2} \quad 0.$$

(6.3)

$$0 \quad \frac{ck}{k_{2,-} \quad 2} \quad 0, \quad (6.1)$$

(2.3) , (6.1) , (6.1) , (6.1) , (6.4)

$$\frac{0}{k} \dots k_2, -2 \quad 0.$$

$$V_j(1, 1) \sim \sum_{2 \rightarrow 1} V_j(1, 2) - V_j(1, 2) \quad (.1)$$

$$\dots (2.4) \dots (5.6)$$

$$\sum_{2 \rightarrow 1} V_j(1, 2) u_1 \Big|_{k \rightarrow 0} 0_k u_1 \Big|_{c_1}$$

$$\dots (1) \dots j \dots (5.5) (5.)$$

$$\sum_{2 \rightarrow 1} \frac{1}{2} V_1(1, 2) \Big|_{k \rightarrow 0} \frac{0_{kk} \frac{k}{2}}{c_1 k / 1} \Big|_{0_k} \frac{0_{kk} \frac{k}{1}}{c_1} \Big|_{c_1} \quad (.2)$$

$$\dots (2) \dots$$

$$\frac{k}{2} (1, 2) \Big|_{1} \frac{k}{1} (1, 2) \quad (.3)$$

$$\dots (2) \dots (2.4) \dots$$

$$\sum_{2 \rightarrow 1}$$

$$s_j(1, 1) \sim u_1 \left| c_1 \right| \frac{1}{3} \frac{j}{1} \frac{c_1}{1} c'_1, \quad \ll 1. \quad (5)$$

$$s_j(1, 1) \sim u_1 \left| c_1 \right| \frac{1}{3} \frac{j}{1} \frac{c_1}{1} c'_1, \quad \ll 1. \quad (5)$$

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$$s_j(1, 1) \sim u_1 \left| c_1 \right| \frac{1}{3} \frac{j}{1} \frac{c_1}{1} c'_1, \quad \ll 1. \quad (5)$$

$$s_j(1, 1) \sim u_1 \left| c_1 \right| \frac{1}{3} \frac{j}{1} \frac{c_1}{1} c'_1, \quad \ll 1. \quad (5)$$

$$s_j(1, 1) \sim u_1 \left| c_1 \right| \frac{1}{3} \frac{j}{1} \frac{c_1}{1} c'_1, \quad \ll 1. \quad (5)$$

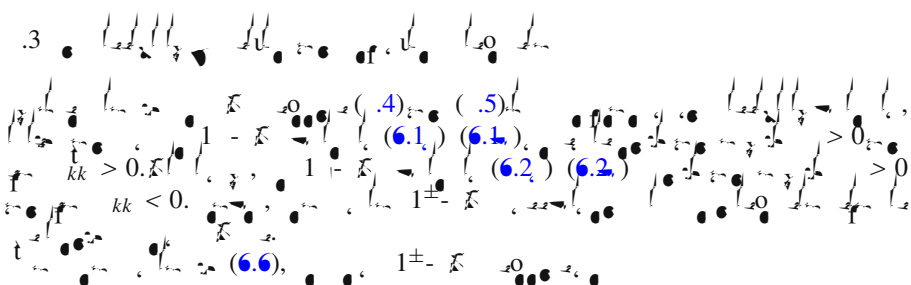
$$s_j(1, 1) \sim u_1 \left| c_1 \right| \frac{1}{3} \frac{j}{1} \frac{c_1}{1} c'_1, \quad \ll 1. \quad (5)$$

$$s_j(1, 1) \sim u_1 \left| c_1 \right| \frac{1}{3} \frac{j}{1} \frac{c_1}{1} c'_1, \quad \ll 1. \quad (5)$$

$$s_j(1, 1) \sim u_1 \left| c_1 \right| \frac{1}{3} \frac{j}{1} \frac{c_1}{1} c'_1, \quad \ll 1. \quad (5)$$

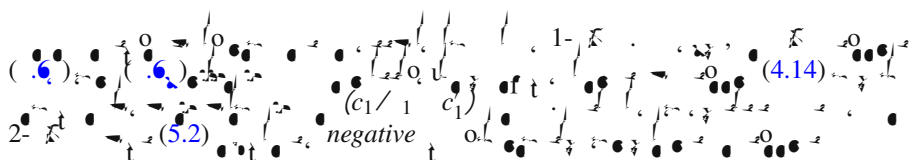
$$s_j(1, 1) \sim u_1 \left| c_1 \right| \frac{1}{3} \frac{j}{1} \frac{c_1}{1} c'_1, \quad \ll 1. \quad (5)$$

1. $kk > 0$, $1 - \delta > 0$

3. 
 Section 3.1 contains musical notation for $kk > 0$ and $1 - \delta > 0$. It includes references to equations (4.4), (4.5), (6.1), (6.2), and (6.6). The notation shows various notes, rests, and accidentals on a staff.

$$s_{\pm}^{(1)}(1, 1) \sim u_1 |c_1| \frac{3 \mp 1}{6} \frac{c_1}{1} c_1', \quad (6)$$

$$v_{\pm}^{(1)}(1, 1) \sim u_1 |c_1| \frac{1 \pm 3}{2} \frac{c_1}{1} c_1', \quad 0 < \ll 1, \quad (6)$$

(6) 
 Section 3.2 contains musical notation for $kk < 0$ and $1 - \delta > 0$. It includes references to equations (5.2), (6.1), (6.2), and (4.14). The notation shows various notes, rests, and accidentals on a staff.

$$s_{\pm}^{(2)}(2, 2) \sim u_2 |c_2| \frac{3 \mp 1}{6} \frac{c_2}{2} c_2',$$

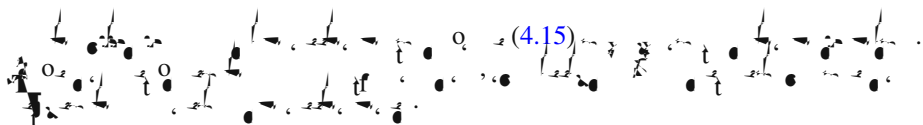
$$v_{\pm}^{(2)}(2, 2) \sim u_2 |c_2| \frac{1 \pm 3}{2} \frac{c_2}{2} c_2', \quad 0 < \ll 1.$$

4. $kk < 0$, $1 - \delta > 0$

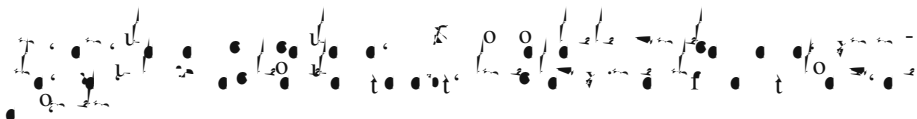

 Section 4.1 contains musical notation for $kk < 0$ and $1 - \delta > 0$. It includes a reference to equation (4.15). The notation shows various notes, rests, and accidentals on a staff.

$$\pm \sim \frac{2 |c_1|}{(1 \mp 1/3)(c_1 |c_1'|)} (M_1 |1|),$$

$$M_{2,\pm} \sim 1 |2^{\pm 1}| (M_1 |1|), \quad 0 < M_1 |1| \ll 1,$$


 Section 4.2 contains musical notation for $kk < 0$ and $1 - \delta > 0$. It includes a reference to equation (4.15). The notation shows various notes, rests, and accidentals on a staff.

5. $kk < 0$, $1 - \delta < 0$


 Section 5.1 contains musical notation for $kk < 0$ and $1 - \delta < 0$. The notation shows various notes, rests, and accidentals on a staff.

$$u = u_1 + u^{(1)}(\cdot, T) + u^{(2)}(\cdot, T) + \dots,$$



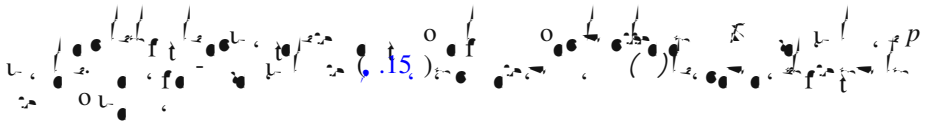
$$\left(- \right) \frac{0(k, -)}{c(-)k} \approx 1 - \frac{k^2}{4c(-)^2} \quad (5)$$

$$(1) \quad 1. \quad (6)$$

$$(5) \quad (5.11)$$

Handwritten musical notation on a staff. The notation includes notes, rests, and dynamic markings. Key annotations include:

- f (forte) and < 0 (piano) dynamic markings.
- Tempo or performance instructions: f (3.), s (slow), and t (tutti).
- Measure numbers in blue: $(.11)$, (6.1) , (6.1) , (6.1) , and (6.1) .
- Other markings: 0 , t , and $() <$.



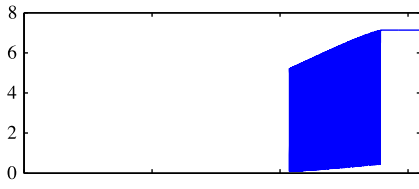
$$() \searrow \frac{p-2}{2p} > \frac{1}{2}, \quad \rightarrow \infty, \quad p > 1, \quad (\cdot 1)$$

$\Gamma \rightarrow \Gamma \cdot \mathbb{Z} \cdot \mathbb{Z}$

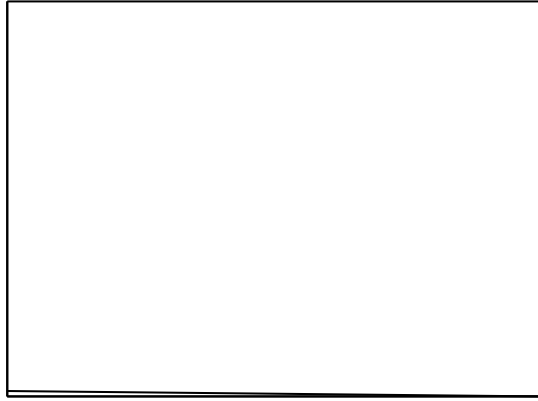
$$0 < p < 1, \quad ()$$



I ...



•





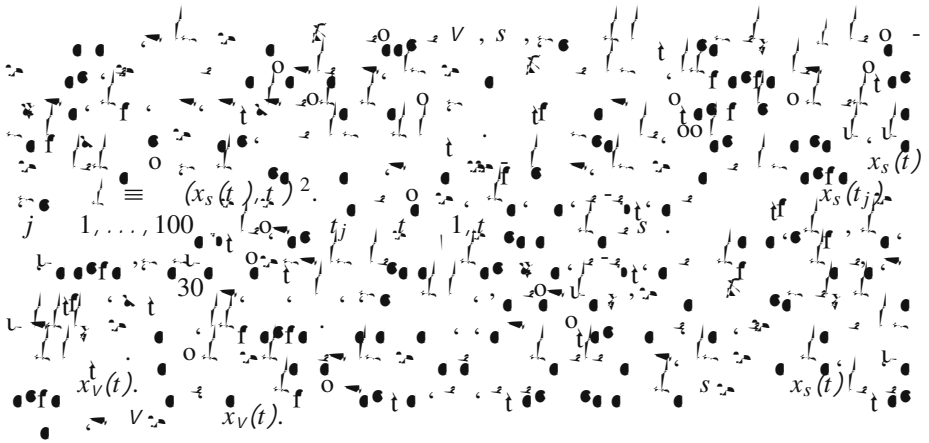
A complex musical score consisting of multiple staves. The notation includes various note values, rests, and dynamic markings such as *ff* and *kk*. There are also some numerical annotations like 3, 4, and 200. A blue circle highlights a specific note in the upper staff. The score is dense and appears to be a transcription of a musical piece.

Acknowledgments

Appendix: Numerical Methods

$$\int_{t_0}^{t_1} f(x, y, t) dt \approx \sum_{i=0}^{n-1} f(x_i, y_i, t_i) \Delta t \quad (3.1)$$

(x,



References

$80(1), 016603 (200)$
 $32(20), 2, 30, 2, 32 (200)$
 (2012)

31(5), 24 422 (200) (1, 4)
 45, oo. 1 6 15 5 6 5 3. 2 0. (200).
 78(1), 013 2 (200)
 41(1), 26 5 (200)
 5(5), 611 613 (1, 64)
 (1, 3)
 36(3), 253 2, 0 (1, 3) 1.
 36(5), 5 1 5, 3 (1, 3) 2.
 36(6), 0 30 (1, 3) 3.
 I. I. 21(10),
 23, 1 240 (200)
 (1, 5) .718,
 524 55 (2013)
 40(3 4), 322 326 (1, 2)
 73(5), 05 602
 (2006)
 (1), 013 61 (200).
 80(4), 043 60 043 60 (200)

(1, 6)

10(3), 53 53, (1, 6)

(1, 3)

24,

20 20, (1, 0)

17(143), 151 (1, 3)

31, 4 4, I (1, 5)

38, 125 155 (1, 5)

68(I), 136 (1,)

3(1), 46 51 (200)

104(), 0 3, 03 (2010)

283, 23 261 (1, 65)

(1, 4)

4, 36, 3, 6 (1, 2)