

# Factorial growth of the number of states in a quantum system

K. L. M. <sup>1,2</sup>, D. B. <sup>1,3</sup>, R. J. L. <sup>1,3</sup>, J. K. <sup>1</sup>, J. G. R. <sup>2</sup>, A. M. R. <sup>1,3</sup>

<sup>1</sup>JILA, NIST, Department of Physics, University of Colorado, Boulder, Colorado 80309, USA

<sup>2</sup>



FIG. 1. ( )  $N$   
 $-1/2$   $R$   
 $= N / 2$

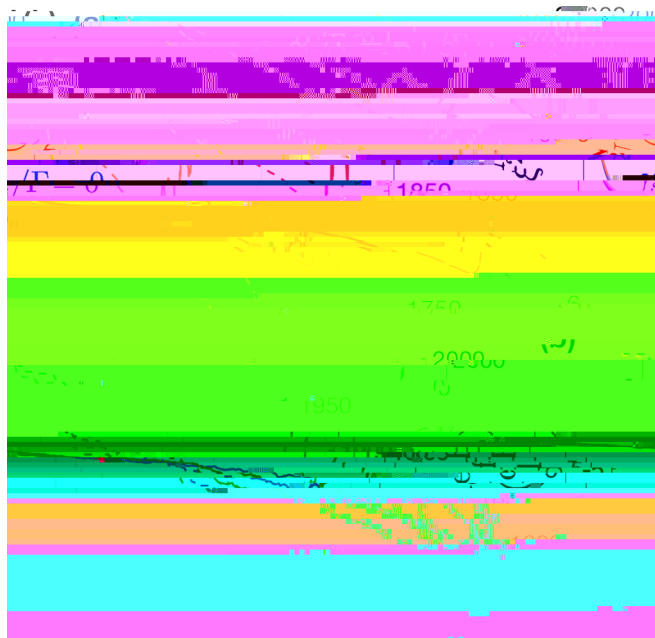
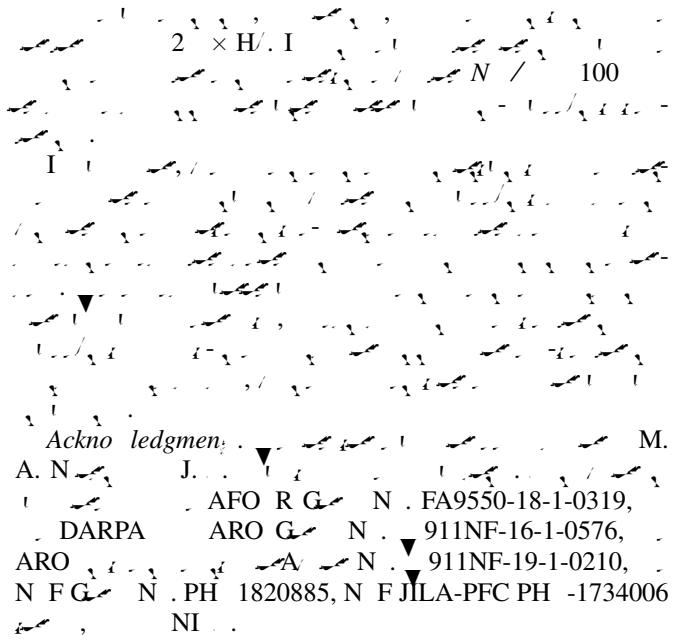
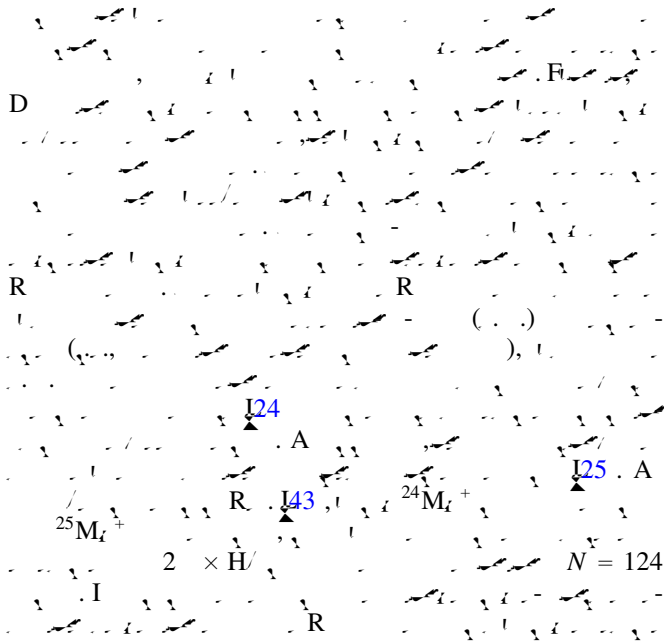


FIG. 2. (a)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2$ . (b)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 4$ . (c)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 8$ . (d)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 16$ . (e)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 32$ . (f)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 64$ . (g)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 128$ . (h)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 256$ . (i)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 512$ . (j)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1024$ . (k)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2048$ . (l)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 4096$ . (m)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 8192$ . (n)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 16384$ . (o)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 32768$ . (p)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 65536$ . (q)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 131072$ . (r)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 262144$ . (s)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 524288$ . (t)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1048576$ . (u)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2097152$ . (v)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 4194304$ . (w)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 8388608$ . (x)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 16777216$ . (y)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 33554432$ . (z)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 67108864$ . (aa)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 134217728$ . (ab)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 268435456$ . (ac)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 536870912$ . (ad)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1073741824$ . (ae)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2147483648$ . (af)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 4294967296$ . (ag)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 8589934592$ . (ah)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 17179869184$ . (ai)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 34359738368$ . (aj)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 68719476736$ . (ak)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 137438953472$ . (al)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 274877906944$ . (am)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 549755813888$ . (an)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1099511627776$ . (ao)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2199023255552$ . (ap)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 4398046511104$ . (aq)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 8796093022208$ . (ar)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 17592186044416$ . (as)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 35184372088832$ . (at)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 70368744177664$ . (au)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 140737488355328$ . (av)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 281474976710656$ . (aw)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 562949953421312$ . (ax)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1125899906842624$ . (ay)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2251799813685248$ . (az)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 4503599627370496$ . (ba)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 9007199254740992$ . (bb)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 18014398509481984$ . (bc)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 36028797018963968$ . (bd)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 72057594037927936$ . (be)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 144115188075855872$ . (bf)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 288230376151711744$ . (bg)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 576460752303423488$ . (bh)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1152921504606846976$ . (bi)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2305843009213693952$ . (bj)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 4611686018427387904$ . (bk)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 9223372036854775808$ . (bl)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 18446744073709551616$ . (bm)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 36893488147419103232$ . (bn)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 73786976294838206464$ . (bo)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 147573952589676412928$ . (bp)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 295147905179352825856$ . (bq)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 590295810358705651712$ . (br)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1180591620717411303424$ . (bs)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2361183241434822606848$ . (bt)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 4722366482869645213696$ . (bu)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 9444732965739290427392$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 18889465931478580854784$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 37778931862957161709568$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 75557863725914323419136$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 151115727451828646838272$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 302231454903657293676544$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 604462909807314587353088$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1208925819614629174706176$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2417851639229258349412352$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 4835703278458516698824704$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 9671406556917033397649408$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 19342813113834066795298816$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 38685626227668133590597632$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 77371252455336267181195264$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 154742504910672534362390528$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 309485009821345068724781056$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 618970019642690137449562112$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1237940039285380274899124224$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2475880078570760549798248448$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 4951760157141521099596496896$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 9903520314283042199192993792$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 19807040628566084398385987584$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 39614081257132168796771975168$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 79228162514264337593543950336$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 158456325028528675187087900672$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 316912650057057350374175801344$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 633825300114114700748351602688$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1267650600228229401496703205376$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2535301200456458802993406410752$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 5070602400912917605986812821504$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 10141204801825835211973625643008$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 20282409603651670423947251286016$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 40564819207303340847894502572032$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 81129638414606681695789005144064$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 162259276829213363391578010288128$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 324518553658426726783156020576256$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 649037107316853453566312041152512$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1298074214633706907132624082305024$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2596148429267413814265248164610048$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 5192296858534827628530496329220096$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 10384593717069655257060992658440192$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 20769187434139310514121985316880384$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 41538374868278621028243970633760768$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 83076749736557242056487941267521536$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 166153499473114484112975882535043072$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 332306998946228968225951765070086144$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 664613997892457936451903530140172288$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1329227995784915872903807060280344576$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2658455991569831745807614120560689152$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 5316911983139663491615228241121378304$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 10633823966279326983230456482242756608$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 21267647932558653966460912964485513216$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 42535295865117307932921825928971026432$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 85070591730234615865843651857942052864$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 170141183460469231731687303715884105728$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 340282366920938463463374607431768211456$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 680564733841876926926749214863536422912$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1361129467683753853853498429727072845824$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2722258935367507707706996859454145691648$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 5444517870735015415413993718908291383296$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 10889035741470030830827987437816582766592$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 21778071482940061661655974875633165533184$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 43556142965880123323311949751266331066368$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 87112285931760246646623899502532662132736$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 174224571863520493293247799005065324265472$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 348449143727040986586495598010130648530944$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 696898287454081973172991196020261297061888$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1393796574908163946345982392040522594123776$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2787593149816327892691964784081045188247552$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 5575186299632655785383929568162090376495104$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 11150372599265311570767859136324180752990208$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 22300745198530623141535718272648361505980416$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 44601490397061246283071436545296723011960832$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 89202980794122492566142873090593446023921664$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 178405961588244985132285746181186892047843328$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 356811923176489970264571492362373784095686656$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 713623846352979940529142984724747568191373312$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1427247692705959881058285969449495136382746624$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2854495385411919762116571938898990272765493248$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 5708990770823839524233143877797980545530986496$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 11417981541647679048466287755595961091061972992$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 22835963083295358096932575511191922182123945984$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 45671926166590716193865151022383844364247891968$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 91343852333181432387730302044767688728495783936$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 182687704666362864775460604089535377456991567872$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 365375409332725729550921208179070754913983135744$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 730750818665451459101842416358141509827966271488$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1461501637330902918203684832716283019655932542976$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 2923003274661805836407369665432566039311865085952$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 5846006549323611672814739330865132078623730171904$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 11692013098647223345629478661730264157247460343808$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 23384026197294446691258957323460528314494920687616$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 46768052394588893382517914646921056628989841375232$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 93536104789177786765035829293842113257979682750464$ . (bv)  $N = 2000$ ,  $\beta = 0$ ,  $\beta_c = 0.9$ ,  $N_c = 1870722095783555735300716585876842265$





*Acknowledgments*. M. A. N. J. is supported by DARPA ARO Grant N. FA9550-18-1-0319, DARPA ARO Grant N. 911NF-16-1-0576, ARO Grant N. 911NF-19-1-0210, NFG Grant N. PH 1820885, NFFJILA-PFC PH -1734006 NI.

[1] L. P. P., A. M. K. O. R. P. R. M. P. **90**, 035005 (2018).  
 [2] *The Physics of Quantum Information*, D. Bouvier, A. E. A. (Cambridge University Press, 2000).  
 [3] B. A. P., *Quantum Information in Continuous Variable* (Springer, 2012).  
 [4] R. J. L., M. A. N., J. R. K. C., J. K.

- [32] P. D. Drell, *Phys. Rev. A* **22**, 662 (1980).
- [33] P. D. Drell, H. J. C. O. C. **27** (1978).
- [34] D. B., R. J. L., J. K., A. M. R.