Observation of Faraday Waves in a Bose-Einstein Condensate

P. Engels* and C. Atherton
Washington State University, Department of Physics and Astronomy, Pullman, Washington 99164, USA

M. A. Hoefer
National Institute of Standards and Technology, Boulder, Colorado 80305, USA
(Received 29 November 2006; published 26 February 2007)

Faraday waves in a cigar-shaped Bose-Einstein condensate are created. It is shown that periodically modulating the *transverse* confinement, and thus the nonlinear interactions in the BEC, excites small amplitude *longitudinal* oscillations through a parametric resonance. It is also demonstrated that even without the presence of a continuous drive, an initial transverse breathing mode excitation of the condensate leads to spontaneous pattern formation in the longitudinal direction. Finally, the effects of strongly driving the transverse breathing mode with large amplitude are investigated. In this case, impact-oscillator behavior and intriguing nonlinear dynamics, including the gradual emergence of multiple longitudinal modes, are observed.

DOI: 10.1103/PhysRevLett.98.095301 PACS numbers: 67.90.+z, 03.75.Kk, 05.45.−a, 47.54.−r

In 1831, Faraday studied the behavior of liquids that are contained in a vessel subjected to oscillatory vertical motion [1]. He found that fluids including alcohol, white of egg, ink, and milk produce regular striations on their surface. These striations oscillate at half the driving frequency and are termed Faraday waves. They are considered to be an important discovery. Since then, the more general topic of pattern formation in driven systems has been met with great interest, and patterns have been observed in hydrodynamic systems, nonlinear optics, oscillatory chemical reactions, and biological media [2].

In this Letter, we study pattern formation by modulating the nonlinearity in a Bose-Einstein condensate (BEC). Nonlinear dynamics arise from the interatomic interactions in this ultracold gas. In the past, the observation of interesting phenomena has motivated researchers to propose and implement various techniques to manipulate the nonlinearity. Such control has been accomplished, for example, by exploiting Feshbach resonances [3]. In our experiment, we investigate an alternative technique, namely, periodically modulating the nonlinearity by changing the radial confinement of an elongated, cigar-shaped BEC held in a magnetic trap. The radial modulation leads to a periodic change of the density of the cloud in time, which is equivalent to a change of the nonlinear interactions and the speed of sound. This can, in turn, lead to the parametric excitation of longitudinal soundlike waves in the direction of weak confinement. This process is analogous to Faraday's experiment where the vertical motion of the vessel produced patterns that were laterally spread out.

It has been shown theoretically that for a BEC, a Faraday type modulation scheme in the case of small driving frequencies leads one to the same type of analysis as would the direct modulation of the interatomic interaction, e.g., by a Feshbach resonance [4,5]. In both cases, the dynamics are governed by a Mathieu equation that is typical for parametrically driven systems. Floquet analysis reveals that a series of resonances exist, consisting of a main resonance at half the driving frequency, and higher resonance tongues at integer multiples of half the driving frequency [4]. From the perspective of phonon number occupation, the Faraday type modulation has recently been analyzed theoretically in [6].

In our experiment we exploit this transverse modulation scheme for three different applications. First, we apply a relatively weak continuous modulation, demonstrate the emergence of longitudinal Faraday waves, and study their behavior as a function of the excitation frequency. Second, we investigate longitudinal patterns that emerge as a consequence of an initial transverse breathing mode excitation without the presence of a continuous drive. This has important consequences in the context of damped BEC oscillations and has been studied theoretically in [7]. Since the first experiments with BECs, the study of collective excitations has been a central theme [8,9]. The transverse breathing mode, which we exploit in our experiments, plays a prominent role: Chevy *et al.* [10] showed that this mode exhibits unusual properties, namely, an extremely high quality factor and a frequency nearly independent of temperature. Finally, in a third set of measurements, we study the situation of a relatively strong modulation, resonantly driving the transverse breathing mode. We show that the condensate responds as an impact oscillator, which leads to intriguing multimode dynamics.

The experiments were carried out in a newly constructed BEC machine that produces cigar-shaped condensates of ^{87}Rb atoms in the $|F = 1; m_F = -1\rangle$ state. The typical atom number in the BEC is 5×10^5, and the atoms are evaporatively cooled until no thermal cloud surrounding the condensate is visible anymore. The atoms are held in a cylindrically symmetric Ioffe-Pritchard type magnetic trap

in the transverse direction was also reproduced in our numerical simulation of the azimuthally symmetric 3D Gross-Pitaevskii equation, displayed as the solid line in Fig. 5. However, the numerics, when starting with a thoroughly relaxed wave function in the initial trap, show a sign of longitudinal pattern formation only after 18 ms, while in the experiment, longitudinal patterns clearly formed already during the third period (9 ms). This, again, hints at the importance of initial noise in the condensate that seeds the parametric amplification. In the experiment, the patterns start out similar to those displayed in Fig. 1 for the case of a weak drive at 321 Hz. But, upon the action of the strong drive, they quickly evolve into more complicated patterns, involving the excitation of several other modes. The inset of Fig. 5 shows an image taken after 5.2 driving periods (a), together with the Fourier transform (b). The Fourier spectrum reveals that several modes corresponding to the first resonance tongue of longitudinal modes with nearly half the driving frequency are excited. In addition, modes at twice the distance from the central Fourier peak are visible. Those modes belong to the second resonance tongue of the main resonance.

In conclusion, we have experimentally observed the effects of parametric resonances in a BEC. The observed resonances lead to Faraday waves along the long BEC axis. These results advance the understanding of collective mode behavior in a condensate, which is one of the key tools to study BEC dynamics. In addition, we have shown that strongly driving the transverse breathing mode leads to an instability whereupon the mode amplitude increases exponentially, accompanied by the strong excitation of multiple soundlike modes.

*