$2\sum_{n=1}^{\infty}\left((b^s a)^2\right)^n$, and so this is a convergent geometric series if $b^s a < 1$. For $s = 1/2$, we have $b^s a = \sqrt{49}/8 = 7/8 < 1$. Since $s \to b^s a$ is a continuous function, it follows that there is some value of $s > 1/2$ for which $b^s a < 1$, and hence we can use the Sobolev embedding theorem to conclude f is continuous.

Alternative solution:

(b) If we also assume that A is a compact operator, prove that M_λ must be finite dimensional

Solution: Note that M_λ is a Hilbert space, so it has some orthonormal basis, and "dimension" means the size of this orthonormal basis. For contradiction, suppose it is not finite dimensional, thus the orthonormal basis is infinite, and one can take a sequence (e_n) of orthonormal basis elements. This is a bounded sequence (since e_n is normalized), so by compactness of A, it means (Ae_n) has a convergent subsequence. But $Ae_n = \lambda e_n$ since e_n is an eigenvector. The sequence (λe_n) is not Cauchy since it is orthogonal ($\lambda e_n - \lambda e_m$ $^2 = \lambda^2 2$ for $n = m$, and recall $\lambda = 0$), and similarly it follows that all subsequences are also not Cauchy, hence there cannot be any convergent subsequence, which is a contradiction.

4.