Department of Applied Mathematics

Preliminary Examination in Numerical Analysis

August 17, 2016, 10 am – 1 pm.

Submit solutions to four (and no more) of the following six problems. Show all your work, and justify all your answers. No calculators allowed.

1. Root finding / Nonlinear equations

Consider the scalar equation $F(x) = 0$. Assume ξ is a root of the equation.

a. Give the recursion for the Newton method for approximating a root.

b. Give conditions on $F(x)$ near ξ that guarantee convergence for x_0

c. If, at the nodes $x = -h, 0, h$, one has function values $y = y_{-h}, y_0, y_h$ s(x) that satisfies the data

x	-1	0	1	2
y	2	3	4	1

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6. Numerical PDE

Consider the parabolic equation

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + f(x,t)$$

where a is a constant.

a. Give the formula for the following finite difference approximations.
   (i) Forward Euler: Centered difference in space, forward difference in time.
   (ii) Backward Euler: Centered difference in space, backward difference in time.
   (iii) Leapfrog: Centered difference in space and centered difference in time.

b. What is the order of accuracy of each method?

c. Use a von Neumann analysis (or any appropriate analysis) to determine the stability of each method.