

Department of Applied Mathematics  
Preliminary Examination in Numerical Analysis  
January 2014

Submit solutions to four (and no more) of the following six problems. Justify all your answers.

1. **Root Finding.**

Construct a continuous function  $f(x)$ , defined over  $x \in (-1; 1)$  such that, for an starting point  $x_0$  that is not itself a root, the Newton iterations for solving  $f(x) = 0$  will be uniquely defined, stay bounded, but nevertheless fail to converge.

2. **Numerical Quadrature.**

The trapezoidal rule has error  $O(h^2)$  and Simpson's rule error  $O(h^4)$ , in both cases with even powers only in their full error expansions. These are the first two members of the Newton-Cotes family of methods, with errors (starting from the trapezoidal case)  $h$  raised to 2, 4, 4,

## 4. Linear Algebra