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(Received 26 February 2009; revised manuscript received 27 April 2009; published 16 July 2009)

Interactions of dispersive shock waves (DSWs) and rarefaction waves (RWs) associated with the Korteweg–de Vries equation are shown to exhibit multiphase dynamics and isolated solitons. There are six canonical cases: one is the interaction of two DSWs that exhibit a transient two-phase solution but evolve to a single-phase DSW for large time; two tend to a DSW with either a small amplitude wave train or a finite number of solitons, which can be determined analytically; two tend to a RW with either a small wave train or a finite number of solitons; finally, one tends to a pure RW.

DOI: [10.1103/PhysRevE.80.016603](https://doi.org/10.1103/PhysRevE.80.016603)

PACS number(s): 05.45.Yv, 47.40.Nm, 52.35.Mw

Shock waves in processes dominated by weak dispersion and nonlinearity have been experimentally observed in plasmas [1], water waves [2], and more recently in Bose-Einstein condensates [3,4] and nonlinear optics [5]; these dispersive shock waves (DSWs) have yielded novel dynamics and in-

Although the (initial) shock front speed is different for DSWs and VSWs ($2v_0/3$ and v_0)

In case IV , a small DSW forms on the left and a large RW forms on the right see Fig. 6 a . As in case II , the front of the DSW interacts with the trailing edge of the RW and decreases the DSW's amplitude and speed. Unlike case II , the front of the DSW does not

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