

# On the finiteness in the deformed Hamiltonian mean-field model

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Abstract: We study the finiteness of the number of periodic orbits in the deformed Hamiltonian mean-field model. We show that the number of periodic orbits is finite for almost all values of the deformation parameter. This result is proved by using the theory of the deformed Hamiltonian mean-field model and the theory of the deformed Hamiltonian mean-field model.

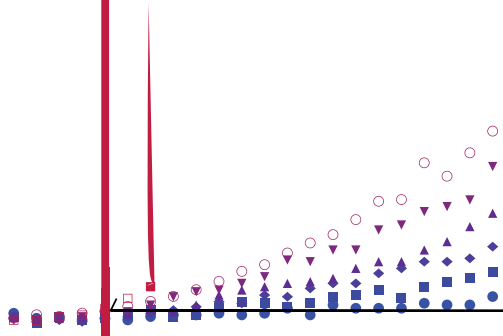
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## I. INTRODUCTION

The Hamiltonian mean-field model (HMF) is a paradigmatic model of a many-body system. It is defined by the Hamiltonian  $H = \sum_{i=1}^N \frac{p_i^2}{2} + \frac{\lambda}{2N} \sum_{i,j=1}^N \cos(\theta_i - \theta_j)$ , where  $\theta_i$  and  $p_i$  are the angle and momentum of the  $i$ -th particle, respectively. The parameter  $\lambda$  is the coupling strength. The HMF model has been extensively studied in the context of statistical mechanics and dynamical systems. In particular, the number of periodic orbits in the HMF model has been a topic of interest for many years. In this paper, we study the finiteness of the number of periodic orbits in the deformed HMF model.

The evolution of the density is given by the continuity equation





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