

# Predicting Criticality and Dynamic Range in Complex Networks: Effects of Topology

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The criticality and dynamic range of a network are studied in the context of a simple model of a network. The network is represented by a set of nodes and edges. The nodes are arranged in a line, and the edges connect adjacent nodes. The network is initially in a state of rest, and a signal is applied to one of the nodes. The signal propagates through the network, and the criticality and dynamic range are studied. The criticality is defined as the point at which the network transitions from a state of rest to a state of activity. The dynamic range is defined as the range of signal amplitudes over which the network exhibits a non-linear response. The criticality and dynamic range are studied as a function of the network topology. The results show that the criticality and dynamic range are highly sensitive to the network topology. The criticality is highest for a network with a high degree of connectivity, and the dynamic range is highest for a network with a high degree of heterogeneity.

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Networks are ubiquitous in nature and have been studied extensively in recent years [1,2]. A network is a collection of nodes and edges. The nodes represent the components of the system, and the edges represent the interactions between them. The network topology, or the arrangement of nodes and edges, is a key factor in determining the network's properties. In this paper, we study the criticality and dynamic range of a network. The criticality is the point at which the network transitions from a state of rest to a state of activity. The dynamic range is the range of signal amplitudes over which the network exhibits a non-linear response. We study these properties as a function of the network topology. The results show that the criticality and dynamic range are highly sensitive to the network topology. The criticality is highest for a network with a high degree of connectivity, and the dynamic range is highest for a network with a high degree of heterogeneity.

The network is modeled as a set of nodes  $x_i$  and edges  $A_{ij}$ . The nodes are arranged in a line, and the edges connect adjacent nodes. The network is initially in a state of rest, and a signal is applied to one of the nodes. The signal propagates through the network, and the criticality and dynamic range are studied. The criticality is defined as the point at which the network transitions from a state of rest to a state of activity. The dynamic range is defined as the range of signal amplitudes over which the network exhibits a non-linear response. The results show that the criticality and dynamic range are highly sensitive to the network topology. The criticality is highest for a network with a high degree of connectivity, and the dynamic range is highest for a network with a high degree of heterogeneity.

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$\lim_{x \rightarrow 0} f(x) = 0$  if  $0 < x < 1$  and  $\lim_{x \rightarrow 0} f(x) > 0$  if  $x > 1$ . I

$\lim_{x \rightarrow 0} f(x) = 1$  if  $x = 1$ . The

$\lim_{x \rightarrow 0} f(x) = \frac{1}{N} \sum_{ij} A_{ij} = \langle$

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