impo an d namical p oce e on comple ne o k a ed on hi fac , e p e en a an i a oec ie cha ac e i a ion of he d namical impo ance of ne o k node and link in e m of hei on he la ge eigenale e ho ho o cha ac e i a ion of he d namical impo ance of node can affec ed deg eedeg ee co ela ion and ne o k commni c e e di c ho o cha ac e i a ion can e ed o op imi e echnie fo con olling ce ain ne o k d namical p oce e appl o e l o eal ne o k

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In recent years, there has been much interest in the study of the structure of networks arising from real world systems, of dynamical processes taking place on networks, and of how network structure impacts such dynamics [1]. The largest eigenvalue of the network adjacency matrix (which we denote) is the key quantity determining a variety of different dynamical processes on networks. For example, (i) for a heterogeneous collection of chaotic and/or periodic dynamical systems coupled by a network of connections, the critical coupling strength [2] for the emergence of coherence is proportional to 1 ; (ii) in a class of percolation problems on directed networks [closely related to the problem of epidemic spreading [3]], the condition for the emergence of a giant component also involves [4]. For other examples where plays a similar role, see Refs. [5–7].

In many situations it might be desirable to control dynamical processes that take place on networks. For example, in epidemic spreading, one would like to increase the threshold for epidemic transmission. In percolation, one might like to identify the key nodes holding the network together and protect them (e.g., in the transportation network or the Internet) or disrupt them (e.g., in the case of a terrorist network or pathogen protein network). Such strategies would greatly benefit from a quantitative characterization of the effect of the removal of the different nodes or edges in the network. We will define the *d namical impo ance* of nodes and edges as the relative change in the largest eigenvalue of the network adjacency matrix upon their removal. This provides an objective quantification of the relative importance of the different elements of the network that could potentially be used to formulate control strategies for those network processes that are governed by the largest eigenvalue of the network adjacency matrix. We also will describe an efficient way to approximate the dynamical importance.

We consider a network as a directed graph with nodes, and we associate to it a $\times$ adjacency matrix whose elements are positive if there is a link going from node to node with $\neq$ and zero otherwise ($\equiv 0$). We denote the largest eigenvalue of by , where $=$ and $=$ with and denoting the right and left eigenvectors of . According to Perron's theorem [7], of all the eigenvalues of , the one with largestki]4c-2(TJ/F5 1 Tf10.6279 0 T

$$(\quad + \quad)(\quad + \quad) = (\quad + \quad)(\quad + \quad) \tag{3}$$

by and neglecting second order terms and , we obtain = . Upon removal of edge → , the perturbation matrix is ( ) = − , and therefore

$$\hat{} \quad = \frac{}{} \tag{4}$$

We now examine the effect of removing node . Upon its removal, the perturbation matrix is given by ( ) = − (+). However, in this case we cannot assume is small as we did before, since = − (the left and right eigenvectors have zero th entry after the removal of node). Therefore, we set = − ∧ , where ∧ is the unit vector for the component, and we assume is8.

Our next example is motivated by the fact that it is sometimes observed that real networks can be subdivided into more or less well defined communities which have different statistics, and thus potentially different dynamical importance. As a simple model of such situation, we specify a division of the nodes in the network into two groups of the same size, and , ($\cup$ $= \{$ A where is the number of removed nodes and ( ) is the largest eigenvalue of the resulting network. We see that using the dynamical importance (solid lines) greatly improves the results over using the degree (short dashed