Department of Applied Mathematics
PROBABILITY AND STATISTICS PRELIMINARY EXAMINATION
January 2020

Instructions:

Do two of three problems in each section (Prob and Stat).	Prob
Place an X on the lines next to the problem numbers	1. ____
that you are NOT submitting for grading.	2. ____
	3. ____
Do not write your name anywhere on this exam.	Stat

(b) What's the probability that after a very long time the machine is working? Explain!

(c) If the machine is currently working, what's the probability it continues doing so without interruptions during the next t units of time? Explain!

(d) If the machine is currently working, what's the probability that it is working t units of time later? Justify!

Problem 3.

Let $r_0, r_1, r_2, \ldots$ be real numbers such that $r_i > 0$, and $\sum_{i=0}^{\infty} r_i = 1$.

Consider a discrete-time homogeneous Markov chain $X = (X_n$

(d) It is well known that Y_1, as the minimum of n exponentials with rate λ, has again an exponential distribution but with rate $n\lambda$. One can show that Y_2 has the same distribution as $E_1 + E_2$, where E_1 and E_2 are independent with $E_1 \quad exp(rate = n\lambda)$ and $E_2 \quad exp(rate = (n-1)\lambda$

(b) Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the distribution with pdf

$$f(x;\theta) = \theta x^{\theta-1} I_{(0,1)}(x).$$

Use part (a) to derive an approximate 100(1 −