The variance of the nondimensional Þeld is $\mathrm{Var}[\ _i(t)] = {}^2$ with

$$ {}^2 = \frac{T}{T_0}, \quad T_0 = \frac{\mu_0 M_s^2 V}{2 k_B}, \tag{5}$$

where k_B is the Boltzmann constant, $V = {}^2_{ex}$ is the characteristic micromagnetic volume, and T_0 is the nondimensional scaling of the absolute temperature T. Table I includes

$$ú = v + \quad , \tag{11b}$$

$$ú = \quad (\quad Š \quad), \tag{11c}$$

$$v́ = \quad {}_{v} v, \tag{11d}$$

where

$$= Š \frac{1}{2} \quad \text{sech}^2 \quad Š \frac{1}{\quad} \quad , \tag{12a}$$

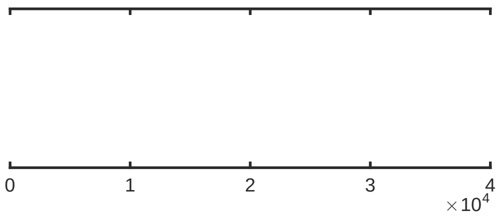
$$= Š h_0 \quad + \quad + \frac{1}{2} \quad \tanh \quad Š \frac{1}{\quad} + 1 \quad , \tag{12b}$$

$$_{v} = Š 2 \quad {}^{2} + \quad Š \quad . \tag{12c}$$

It is necessary to carefully choose parameters so that this Þxed point is stable, i.e., so that all eigenvalues in Eq. (12) are negative. The condition $> h_0$ is sufÞcient for $\, , \; < 0$, but in order to ensure that $_{v} < 0$, we require additionally that

$$(2 \quad + h_0) > \frac{1}{2} \quad \tanh \quad Š \frac{1}{\quad} + 1 \quad . \tag{13}$$

Note that the inequality requirement for stability in (13) was not identiÞed previously [21], and is essential to understanding the dynamics of the droplet. It is possible to visualize the region of linear stability in the $(h_0, \; / \;)$ plane as in Fig. 2. The left


0
1
2
3
4
×10^4

plitude dynamics, respectively, of spatially uniform STOs [24]. For the linearized system, the resulting generation linewidth is linearly dependent on temperature, whereas the nonlinear system exhibits a linewidth enhancement when approaching room temperature, reßecting the coupling between the dropletÕs constituent variables. Full-scale micromagnetic simulation, including the fully nonlinear spatial variation of the system, qualitatively agree with the numerical results. However, we do not observe convergence toward the linear theory at low temperatures using a standard micromagnetic package [30]. This suggests the study of droplet generation linewidth as a test problem for stochastic micromagnetic codes [33].

The analytical and numerical linewidths obtained are two orders of magnitude below the typical linewidths observed in experiments. This disagreement may be caused by the small NC radii used experimentally, the existence of nonlocal dipolar and current-induced Oersted Þelds, and the aforementioned drift instabilities for data-acquisition timescales. In fact, micromagnetic simulations performed with a radius similar to those experimentally fabricated to date return linewidths in the same order of magnitude when both nonlocal and current-induced Oersted Þelds are included. The relevance of such Þelds in the generation linewidth motivates their inclusion in the analytical theory. For thin Þlms, the effect of nonlocal dipole Þelds on deterministic droplet dynamics has been shown

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