

# META-ANALYSIS

## A Case Study

David G. Cook

University of Colorado, Boulder

*This article raises some questions about the usefulness of meta-analysis as a means of reviewing quantitative research in the social sciences. When a meta-analytic model for SAT coaching is used to predict results from future studies, the amount of prediction error is quite large. Interpretations of meta-analytic regressions and quantifications of program and study characteristics are shown to be equivocal. The match between the assumptions of the meta-analytic model and the data from SAT coaching studies is not good, making statistical inferences problematic. Researcher subjectivity is no less problematic in the context of a meta-analysis than in a narrative review.*

**Keywords:** meta-analysis; literature review; SAT coaching; statistical inference

Meta-Analysis in Social Research (1981), Statistical Methods for Meta-Analysis (1985), 1980, meta-analysis (1980), 2003, 1,000, Experimental and Quasi-Experimental Design for Generalized Causal Inference,

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20 (R. G. G., C. I.,  
C. I. 2002, 446).

(1986; 1988; B. 2004; B. 2003).

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TABLE 1: Observed and Predicted Effects From New Coaching Studies

| Re            | S d   | C ac | Effic | P ed c ed C ac |        | Effic F |        | Bec e (1990) |        |
|---------------|-------|------|-------|----------------|--------|---------|--------|--------------|--------|
|               |       |      |       | M de A         | M de B | M de B  | M de C | M de C       | M de D |
| H e e (1984)  | SAT-V | 57   |       | 30             | 11.6   | 12.9    | 24.5   |              |        |
| F a e (1987)  | SAT-M | 37   |       | 30             | 25.5   | 1.2     | 35.8   |              |        |
|               | SAT-V | 16   |       | 30             | 11.6   | 1.9     | 0.8    |              |        |
| H a e (1988)  | SAT-M | 16   |       | 30             | 25.5   | 13.6    | 12.1   |              |        |
|               | SAT-M | 21   |       | 30             | 25.5   | 14.5    | 8.1    |              |        |
| W a (1988)    | SAT-V | 11   |       | 30             | 11.6   | 2.7     | 0.5    |              |        |
|               | SAT-M | 16   |       | 30             | 25.5   | 14.4    | 11.8   |              |        |
| S edec (1989) | SAT-V | 0    |       | 30             | 11.6   | 2.7     | 0.2    |              |        |

TABLE 2: Average Prediction Error From Becker's (1990) Meta-Analytic Models

| Model   | Mean Error | SD   |
|---------|------------|------|
| Model A | 0.17       | 0.09 |
| Model B | 0.17       | 0.09 |
| Model C | 0.17       | 0.09 |
| Model D | 0.17       | 0.09 |

### INTERPRETING META-ANALYTIC REGRESSIONS AND QUANTIFYING STUDIES

#### INTERPRETING META-ANALYTIC REGRESSIONS

Becker (1990) (p. 393). In a meta-analysis of 100 studies, the average prediction error was 0.17 (SD = 0.09). The average prediction error for Model A was 0.17 (SD = 0.09). The average prediction error for Model B was 0.17 (SD = 0.09). The average prediction error for Model C was 0.17 (SD = 0.09). The average prediction error for Model D was 0.17 (SD = 0.09).

11)  $\int_0^1 \frac{1}{1+x^2} dx = \arctan(1) - \arctan(0) = \frac{\pi}{4}$   
12)  $\int_0^1 \frac{1}{1+x^2} dx = \arctan(1) - \arctan(0) = \frac{\pi}{4}$   
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19)  $\int_0^1 \frac{1}{1+x^2} dx = \arctan(1) - \arctan(0) = \frac{\pi}{4}$   
20)  $\int_0^1 \frac{1}{1+x^2} dx = \arctan(1) - \arctan(0) = \frac{\pi}{4}$







**TABLE 4: Estimated Coaching Effects in Randomized Studies**

| Re a d S d            | SAT-M | SAT-V |
|-----------------------|-------|-------|
| Ade a a d P e (1980)  |       |       |
| Sc A                  |       | 22    |
| Sc B                  |       | 9     |
| Sc C                  |       | 14    |
| Sc D                  |       | 14    |
| Sc E                  |       | 1     |
| Sc F                  |       | 14    |
| Sc G                  |       | 18    |
| Sc H                  |       | 1     |
| E a a d P e (1973)    |       |       |
| G A                   | 12    |       |
| G B                   | 25    |       |
| G C                   | 11    |       |
| La c e e (1985)       | 8     | 0     |
| R be a d O e e (1966) |       |       |
| Sc A                  |       | 17    |
| Sc B                  | 12    |       |
| Z a (1988)            | 51    | 14    |
| Med a effec e a e     | 12    | 14    |

**TABLE 5:**

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8 ... 30  
 ( ... ) ... 30  
 ... D ... 30

Handwritten text, possibly a signature or a name, written in a cursive script.





11-17  
17  
556  
1,566  
(Gill, A.)  
(Baker, 1990, 397).  
(Baker, 1982).  
Gill, A.

*Calculating effect sizes for meta-analytic regressions.*  
Baker, 1990  
Baker, 1982  
Gill, A.

(1980) 17  
B  
55  
17  
B  
A  
to (2). A  
B  
(1979)  
A  
A  
to B  
to L 1965; 1961; C 1967; C 1987;  
1984). 13  
( $g_{hi}$ ),  
( $g_{hi}$ ). 13 70.  
B  
11  
13  
1960 1967  
(L 1965;  
1961; C 1967):

(1961)  
D.C.





The image shows a complex musical score with multiple staves. The notation is dense and overlapping, featuring various musical symbols such as notes, clefs, and dynamic markings. The text is partially obscured by the musical notation and is difficult to read. Some legible fragments include "B", "11", "12", "3-5", and "RA, I".

72-636)-63(\*)5(8(\*a.)-1.670.5( 23 1 33)

Let  $A = \{1, 2, \dots, h\}$  and  $B = \{1, 2, \dots, t\}$ . Let  $X_{hij}$  and  $Y_{hij}$  be the random variables representing the number of observations in the  $i$ th stratum of the  $j$ th treatment group.

Assume that

$$X_{hij} \sim N(\mu_{hi}^C, \sigma_{hi}^2) \text{ and } Y_{hij} \sim N(\nu_{hi}^C, \sigma_{hi}^2),$$

where  $\mu_{hi}^C$  and  $\nu_{hi}^C$  are the means and  $\sigma_{hi}^2$  is the variance of the  $i$ th stratum of the  $j$ th treatment group. Let  $X_{hi} = (X_{hij})_{j=1,2,\dots,t}$  and  $Y_{hi} = (Y_{hij})_{j=1,2,\dots,t}$  be the vectors of random variables representing the number of observations in the  $i$ th stratum. Let  $X_{hi} \sim N(\mu_{hi}^C, \Sigma_{hi}^C)$  and  $Y_{hi} \sim N(\nu_{hi}^C, \Sigma_{hi}^C)$  be the multivariate normal distributions of the vectors of random variables representing the number of observations in the  $i$ th stratum. Let  $X_{hi}^U$  and  $Y_{hi}^U$  be the vectors of random variables representing the number of observations in the  $i$ th stratum of the  $j$ th treatment group in the  $U$ th population. Let  $X_{hi}^U \sim N(\mu_{hi}^U, \Sigma_{hi}^U)$  and  $Y_{hi}^U \sim N(\nu_{hi}^U, \Sigma_{hi}^U)$  be the multivariate normal distributions of the vectors of random variables representing the number of observations in the  $i$ th stratum of the  $U$ th population.

$$X_{hij}^C \sim N(\mu_{hi}^C, \sigma_{hi}^2) \text{ and } Y_{hij}^C \sim N(\nu_{hi}^C, \sigma_{hi}^2), \quad (4)$$

where

$$X_{hij}^U \sim N(\mu_{hi}^U, \sigma_{hi}^2) \text{ and } Y_{hij}^U \sim N(\nu_{hi}^U, \sigma_{hi}^2). \quad (5)$$

Let  $\sigma_{hi}^2$  be the variance of the  $i$ th stratum of the  $j$ th treatment group in the  $C$ th population. Let  $\mu_{hi}^C, \nu_{hi}^C, \mu_{hi}^U, \nu_{hi}^U$  be the means of the  $i$ th stratum of the  $j$ th treatment group in the  $C$ th and  $U$ th populations. Let  $\Sigma_{hi}^C$  and  $\Sigma_{hi}^U$  be the covariance matrices of the vectors of random variables representing the number of observations in the  $i$ th stratum of the  $C$ th and  $U$ th populations. Let  $\mu_{hi}^C, \nu_{hi}^C, \mu_{hi}^U, \nu_{hi}^U$  be the means of the  $i$ th stratum of the  $j$ th treatment group in the  $C$ th and  $U$ th populations. Let  $\Sigma_{hi}^C$  and  $\Sigma_{hi}^U$  be the covariance matrices of the vectors of random variables representing the number of observations in the  $i$ th stratum of the  $C$ th and  $U$ th populations.



1. (A) 1980  
1980, 1961; L. 1965;  
1973; C. B. 1978; 1980;  
1980).

C. 1980  
B. 1980  
1980

... B... (2003) ...  
... 50% ...  
... to p  
... (all ...  
... ) ...  
... t ...

to  
B  
C  
(1978), (1980), (1981),  
C (1982), B (1989).  
90 -251.93

TABLE 6: Studies by Coaching Mode and Design

| Cac | T e    | Me d ca De   |  |  |
|-----|--------|--|--|--|
|     |        | Ra d ed C  | Obe a a C  | N C  |
| Sc  | -ba ed | R be a d O e e (1966)<br>E a a d P e (1973)<br>A de a a d P e (1980)<br>S a (1992) | D e (1953)<br>F e c (1955)<br>Dea (1958)<br>Keefa e (1976)<br>K c (1979)<br>J (Sa Fa c c e) (1984) <sup>a</sup><br>B e (1986)<br>R e d a d Obe a (1987)<br>H a e (1988)<br>W , C d , a d Ma e (1989)<br>S c e d e (1992)<br>W e (1996) | Pa e (1961)<br>Ma (1965)<br>J (A a a, Ne e) (1984) <sup>a</sup><br>Y |



|   |            |                             |              |
|---|------------|-----------------------------|--------------|
| C | e ca-ba ed | F a e (1960)                | Ka a (2002)  |
|   |            | W a (1962)                  |              |
|   |            | Fede a T ad g C             |              |
|   |            | B Re a Office (1978)        | d a d ea a e |
|   |            | B ea f C e P ec (1979)      |              |
|   |            | R c (1980)                  |              |
|   |            | S d (1980)                  |              |
|   |            | Se , Be a d, a d K a (1982) |              |
|   |            | F a e (1987)                |              |
|   |            | W a (1988)                  |              |
|   |            | Z a (1988) <sup>a</sup>     |              |
|   |            | S edec (1989)               |              |
|   |            | S (1989)                    |              |
|   |            | S (1990)                    |              |
|   |            | P a d R c (1999)            |              |
|   |            | B (2001)                    |              |
| C | e -ba ed   | H ee (1984)                 |              |
|   |            | La c ee (1985)              |              |

• It is a common mistake to think that the only way to improve your credit score is to pay off your debts. While this is certainly a good idea, it is not the only way. You can also improve your credit score by paying your bills on time, keeping your credit utilization low, and checking your credit report for errors.







- All people to the left of the line will be elected.
- All people to the right of the line will be elected.












(2000) ...  
 ...  
 ...  
 ... A.C. (1982, 108)

**APPENDIX A**  
**SAT Academic Reading, 1953-2001**

| Sample Academic Text | Grade | Year ( ) | SES (b.9998 -7.9998 1.7004 79484019916 269) |
|----------------------|-------|----------|---|
|----------------------|-------|----------|---|





|  |           |           |         |  |   |           |   |
|--|-----------|-----------|---------|--|---|-----------|---|
| Z a (  -SES<br>a e) (1988)  | 21/55     | 21/55     | 11      | M e b c ( ba )   | NY  | 1985-1986 | H  |
| S (1989)   | 200/438   | 200/438   | 12      | 8 a e ( b ba )   | MD, D.C.  | 1987-1988 | H  |
| S edec (1989)  | 264/535   | 264/535   | 12      | 10 b c a d a e   | PA  | 1988-1989 | H  |
| S (1990)   | 631/1,132 | 631/1,132 | 12      | 14 a e ( b ba )  | MD, NJ  | 1989      | H  |
| P e a d R c (1999)   | 427/2,086 | 427/2,086 | 11 , 12 | M e b c a d<br>a e   | USA   | 1995-1996 | M ed  |
| B  (2001)   | 503/3,144 | 503/3,144 | 11 , 12 | M e b c a d<br>a e   | USA   | 1991-1992 | M ed  |
| Ra d ed de <br>Sc -ba ed c ac <br>R be a d O e e<br>(1966) | 154/265   | 188/310   | 12      | 18 b c (a Bac ,<br>ba , a d a )<br>12 b c ( ba<br>a d b ba ) | TN  | 1965      | L   |
| E a a d P e (1973)   | NA        | 288/417   | 11      |  | NJ, OH, PA  | 1970-1971 | M ed  |
| Ade a a d P e<br>(1980)  | 239/559   | NA        | 11      | 8 b c a d a e  | 7 Neg<br>E  a d<br>a e | 1977-1978 | M ed  |
| J (Sa Fa c c<br>e) (1984)  | 23/35     | 23/35     | 11      | M e b c (a<br>Bac , ba )                                     | CA  | 1983-1994 | L   |
| S a (1992) <br>C e c a c ac                            | 61/122    | 61/122    | 12      | 3 b c ( b ba )   | CA  | 1988      | M ed  |
| Z a ( -SES a e )<br>(1988)   | 16/33     | 16/33     | 11      | M e b c ( ba )   | NY  | 1985-1986 | L   |

(c ed)



**APPENDIX B**  
**C. d. Tea. e. C a a c e . . c . f Ne. S i d e**

| S d                   | G a d<br>Mea | SAT-M | G | D   | VI | MI | AI | IP | TP | TS | OA | HW | CI | WC | AC |
|-----------------------|--------------|-------|---|-----|----|----|----|----|----|----|----|----|----|----|----|
| H e e                 | 1            | 1     | 1 | 3.5 | 1  | 1  | 1  | 1  | 0  | 1  | 0  | 0  | 1  | 0  | 0  |
| F a e                 | 1            | 1     | 1 | 15  | 1  | 1  | 0  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| H a e                 | 1            | 1     | 1 | 4   | 0  | 1  | 1  | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 1  |
| W a                   | 1            | 1     | 1 | 15  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| S e d e c             | 1            | 1     | 1 | 15  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| W . , C d , a d M a e | 1            | 1     | 1 | 15  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| S                     | 1            | 1     | 1 | 15  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| S a                   | 1            | 1     | 1 | 4   | 1  | 1  | 0  | 0  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| S c e d e             | 1            | 1     | 1 | 16  | 0  | 1  | 2  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| H e a d K e f f e     | 1            | 0     | 1 | 8   | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  |
| W e                   | 1            | 0     | 1 | 68  | 1  | 0  | 2  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| P e a d R c           | 1            | 1     | 1 | 15  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| B . .                 | 1            | 1     | 1 | 15  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| K a Yea 1             | 1            | 1     | 0 | 30  | 0  | 1  | 2  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  |
| K a Yea 2             | 1            | 1     | 0 | 30  | 0  | 1  | 2  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  |

NOTE: D=d a f c a c . (b d a e a e b e e e d a B e c e ' [1990] e e ), V I = e b a c , M I = a c -  
 , A I = a a c , I P = e a c c e , T P = e a c c e , T S = e - a . , O A = e a c e , W C = a - c , A C = a e a -  
 e c .

**APPENDIX C**  
**C. d. De. . . C a a c e . . . c . . f Ne. S, d e.**

| S d               | Yea | P b | Ma c | Ra d | ETS | Se | V |
|-------------------|-----|-----|------|------|-----|----|---|
| H ee              | 82  | 0   | 0    | 1    | 0   | 1  | 2 |
| Fa e              | 87  | 0   | 0    | 0    | 0   | 2  | 2 |
| Ha e              | 88  | 0   | 0    | 0    | 0   | 1  | 2 |
| Wa                | 88  | 1   | 0    | 0    | 0   | 2  | 2 |
| Selec             | 89  | 1   | 0    | 0    | 0   | 2  | 2 |
| W, C, d, a d Ma e | 89  | 0   | 0    | 0    | 0   | 2  | 2 |
| S                 | 90  | 1   | 0    | 0    | 0   | 2  | 2 |
| S a               | 92  | 0   | 0    | 1    | 0   | 1  | 2 |
| Sc ede            | 92  | 0   | 0    | 0    | 0   | 2  | 2 |
| H e a d Keffe     | 95  | 1   | 0    | 1    | 0   | 2  | 2 |
| W e               | 96  | 0   | 1    | 0    | 0   | 2  | 2 |
| P a d R c         | 99  | 1   | 0    | 0    | 1   | 1  | 2 |
| B                 | 101 | 1   | 0    | 0    | 0   | 1  | 2 |
| Ka a Yea 1        | 101 | 1   | 0    | 0    | 0   | 2  | 2 |
| Ka a Yea 2        | 101 | 1   | 0    | 0    | 0   | 2  | 2 |

NOTES

1. ... (1976). ... (1987). ...
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4. ... (1999). ... (1989). ... 1990
5. B... (1989). ...
6. D... B... (1989). ...
7. ... D... C...
8. ... (1965).
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10. ... D... (1953), ... (1955), D... (1958), ... (1988). ... A... (1980).
11. ... (1985).
12. ... B... (2004, ... 4).
13. ... (1987). ...

REFERENCES

A... D... D... 1980. ... *American Educational Research Journal* 17:239-53.

A... A... 1981. ... *American Psychologist* 36 (10): 1086-93.

- Bollen, K. A. B. 1988. *Structural Equations with Latent Variables*. New York: Wiley.
- . 1990. *Measurement Error in Models with Latent Variables: A Review of Educational Research* 60 (3): 373-417.
- Bollen, K. A. 2004. *Regression analysis: A constructive critique*. Thousand Oaks, CA: Sage.
- Bollen, K. A., & D. A. Stoker. 2003. *Law, punishment, and social control: Essays in honor of Sheldon Messinger*. *Journal of Quantitative Criminology*, 235-54. doi:10.1007/s11116-003-9000-2.
- Bollen, K. A. 1989. *Latent variables in educational measurement*. *Journal of Educational Measurement*, 429-44. doi:10.1111/j.1745-1222.1989.tb00111.x.
- Bollen, K. A., & A. R. L. Bollen. 1987. *Writer's guide: Psychology*. Los Angeles: D. C. Heath.
- Bollen, K. A., & D. C. 2001. *Measurement error in structural equation models: A review of the literature*. *Chance* 14 (1): 10-18.
- . 2002. *Measurement error in structural equation models: A review of the literature*. *Chance* 15 (1): 7-8.
- . 2002. *Measurement error in structural equation models: A review of the literature*. *Chance* 15 (1): 7-8.
- . 2004. *Measurement error in structural equation models: A review of the literature*. *Journal of Educational and Behavioral Statistics* 29 (4): 397-420.
- . 2004. *Measurement error in structural equation models: A review of the literature*. *Rethinking the SAT: Perspectives based on the November 2001 conference at the University of California, Santa Barbara*. *Journal of Educational Measurement*, 217-34. doi:10.1111/j.1745-1222.2004.tb00111.x.
- Bollen, K. A., & B. 1986. *Measurement error in structural equation models: A review of the literature*. *Journal of Educational Measurement*, 217-34. doi:10.1111/j.1745-1222.1986.tb00111.x.
- Bollen, K. A., & D. C. 1987. *Measurement error in structural equation models: A review of the literature*. *Journal of Educational Measurement*, 217-34. doi:10.1111/j.1745-1222.1987.tb00111.x.
- Cronbach, L. J. 1987. *Measurement error in structural equation models: A review of the literature*. *Journal of Educational Measurement*, 217-34. doi:10.1111/j.1745-1222.1987.tb00111.x.
- Cronbach, L. J. 1967. *Measurement error in structural equation models: A review of the literature*. *Journal of Educational Measurement*, 217-34. doi:10.1111/j.1745-1222.1967.tb00111.x.
- Personnel and Guidance. 1967. *Measurement error in structural equation models: A review of the literature*. *Journal of Educational Measurement*, 217-34. doi:10.1111/j.1745-1222.1967.tb00111.x.

Gill, B. C. L. (1979). *Effects of coaching standardized admission examinations: Revised statistical analyses of data gathered by the Boston Regional Office of the Federal Trade Commission*. Washington, DC: U.S. Government Printing Office.

Gill, B. C. L. (1987). *The Princeton Review*. The Newsletter. D. Phillips, A. D. Phillips, & J. A. Phillips.

Gill, B. C. L. (1960). *Personnel and Guidance Journal* 38:713-19.

Gill, B. C. L. (1955). *The coach ability of the SAT in public schools*. *Personnel and Guidance Journal* 3:1-5.

Gill, B. C. L. (2000). *Journal of Educational Research* 93:25-31.

Gill, B. C. L. (1981). *Meta-analysis in social research*. Beverly Hills, CA: Sage.

Gill, B. C. L. (1988). *Journal of Educational Research* 81:1-5.

Gill, B. C. L. (1990). *The future of meta-analysis*. *Journal of Educational Research* 83:11-20.

Gill, B. C. L. (1985). *Statistical method for meta-analysis*. *Journal of Educational Research* 78:3-10.

Gill, B. C. L. (1995). *Journal of Educational Research* 88:492-3.

07TTD [( ) -

———. 1982. *The Educational Psychologist* 17 (2): 67-91.

L. A. 1981. *Psychological Bulletin* 89:191-216.

———. 1986.



1988. *Journal of Applied Econometrics*, 3(1), 1-11.

*Derek C. Briggs is an assistant professor specializing in quantitative methods and policy analy-*