o o o (2016) 15.25 27

 0.025 3 (10.1007/ $/01037$ -015-064-y) / o \downarrow 1 / o \downarrow 2 470.302 537 .6352 563 1.35 672 .34 0.025 3 (10.1007/ 01037-015-06 44) dividend and 0.025 37 (3882 5631.35 672 .34

$$
2.1 \qquad \qquad f \qquad \qquad
$$

2.1
\n
$$
\begin{array}{cccccccc}\n\mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf
$$

 $\begin{pmatrix} r & 0 \\ r & 0 \end{pmatrix}$ undergoing isotropic growth via the secretion, transport and $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$ of extra-cellular matrix $\begin{pmatrix} 0 & 0 \end{pmatrix}$ of $\begin{pmatrix} 0 & 0 \end{pmatrix}$ molecules with $\begin{pmatrix} 0 & 0 \end{pmatrix}$ an existing hydrated porous structure. Let us further assume that the characteristic length scale of θ each constituent of each constituent of each constituent of each constituent of θ p ore since \mathbb{R} in \mathbb{R} in \mathbb{R} \mathbb tissue so that the latter can be viewed as a continuum, and α in α deformable α **e** Ω_0 , in $\mathbf{r} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathbf{r} & 0 \end{pmatrix}$ in an appropriate Cartesian coordinate $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is assumed by $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ \mathcal{Q} oⁿ of \mathcal{Q} of \mathcal{Q} or \mathcal{Q} or characterized by the integration of \downarrow \sim 0 \cdot \downarrow \sim \cdot \cdot \downarrow \sim number of moles of α−constituent per reference volume of $\frac{1}{\sqrt{2}}$ mixture. One can alternatively use the normal volume $\frac{1}{\sqrt{2}}$ volume fraction $\frac{1}{\sqrt{2}}$ $\begin{array}{ccccccccc}\n\mathcal{A} & \mathcal{A} &$ $\begin{array}{ccc}\n\uparrow & & & \\
\uparrow & & & \\
\downarrow & & & \n\end{array}$

$$
\Phi_{\alpha} \qquad \qquad \nu_{\alpha}^* \qquad \qquad \nu_{\alpha} \qquad \qquad (1)
$$

where να is the true molar volume of the constituent. For simplicity, we assume here that all constituents are made of incompressible matter, that is, the specific (molar) volume να does not change over time, regardless of their solid or fluid state. Note that this concept is to be understood within the context of homogenized equivalent constituents which enables us to avoid complications associated with incompressibl[e](#page-19-3) [constituents](#page-19-3) [in](#page-19-3) [their](#page-19-3) [natural](#page-19-3) [state](#page-19-3) [\(](#page-19-3)Humphrey and Rajagopal [2002\)](#page-19-3). To characterize the physical state of the mixture, it is convenient to decompose these volume fractions into contributions from solid and fluid phases. Omitting the arguments (, *t*) for clarity, we write *n* ^α ¹ ^α and *ⁿ* ^α ¹ α where a superscript denotes the solid state, while no superscripts are used for fluid phases. Assuming that the mixture is saturated, it can be shown that the nominal volume fractions must verify at any time: *n*

 α 1 Φ_{α}^{\star} Φ_{α} (2)

$$
\begin{array}{ccccccccc}\n0 & 0 & \overline{11} & \overline{1} & 0 & 0 & \overline{1} & \overline{1} & \overline{1
$$

k is the Boltzmann constant and Φ

 $4.2.26$ Transport

 \mathbf{m} , \mathbf{m} at constant \mathbf{m} at constant volume. However, when the constant volume \mathbf{m} **i**₂ - ^{*o*} *α c*^α *Ω*

$$
\begin{array}{ccccccccc}\n\mathfrak{P}_{\mathsf{T}} & \mathfrak{v} & \mathfrak{t}_{1} & \mathfrak{a}_{1} & \mathfrak{v} & \mathfrak{t}_{2} & \mathfrak{t}_{1} \\
\mathfrak{P}_{\mathsf{T}} & \mathfrak{v} & \mathfrak{v} & \mathfrak{t}_{1} & \mathfrak{t}_{1} & \mathfrak{t}_{1} & \mathfrak{t}_{1} & \mathfrak{t}_{1} \\
\mathfrak{p} & \mathfrak{t}_{1} \\
\mathfrak{p} & \mathfrak{p} \\
\mathfrak{p} & \mathfrak{p} \\
\mathfrak{p} & \mathfrak{p} \\
\mathfrak{p} & \mathfrak{p} \\
\mathfrak{p} & \mathfrak{p} \\
\mathfrak{p} & \mathfrak{p} \\
\mathfrak{p} & \mathfrak{p} &
$$

4.4 Elastic potentials

In Eq. [\(15\)](#page-5-0), it was shown that the elastic energy stored of a growing body can be additively decomposed into contributions from its different solid phases, all possessing their own potential energy depending on deformations and mechanical properties. This decomposition requires the definition of elastic potentials ^α, defined per unit reference volume of pure constituent; two examples are given here for our polymer matrix system.

 $4.4.1$ μ ² μ \rightarrow μ ² μ ² μ ²

Following the classical Flory-Rehner model for the elastic energy of a hydrogel [\(Flory 1953\)](#page-19-6), the elastic potential of the polymer network, with reference in its dry state, reads:

$$
s_{\bullet} = s_{\bullet}^{\prime} \qquad \frac{s_{\bullet}}{v_{s_{\bullet}}} = \frac{a_{\bullet}}{v_{s_{\bullet}}} = \frac{a_{\bullet}}{v_{s_{\bullet}}} \qquad \frac{f_{\bullet}}{v_{s_{\bullet}}} \tag{45}
$$

Here represents the molar ratio of cross-links and polymer chains, while ν ^p is the molar volume of the polymer. We also note that the Green–Lagrange strain *^d* ^p is measured with respect to the dry (or unswollen) polymer state; this motivates the use of the upper-script *d*

constituent at the time of deposition on the growth dynamics of a biological tissue. We here propose to emphasize its role on both growth and the buildup of residual stresses within the material. Referring to Sect. [4.3,](#page-8-0) we consider three main types of depositions: • Case 1: The deposition strain *J* is independent of the time of deposition. In this context, we investigate situations in which the matrix deposits in (a) a stress-free configuration, i.e.,

 φ (1) o.

 $\frac{2}{\partial}$ $\frac{2}{\sigma}$ **ex**

∂ *R R*

$$
2.1 \quad \bullet \quad \bullet \quad \bullet
$$

5.2.2 Mechanical equilibrium and residual stresses

 $0 \t0 \t0 \t0 \t1 \t0 \t1$, $0 \t-$

Using a spherical coordinate system (, ,) and assuming central symmetry, it can be found that displacements only occur in the radial direction (Fig. [\)](#page-15-0). Denoting by the coordinate of a point that was originally in , one can derive the following expression for the deformation gradient:

$$
\frac{\partial}{\partial y} = -1 \tag{5}
$$

$$
\begin{array}{ccccccccc}\n & \alpha & & \frac{1}{4} & & \frac{1}{4}
$$

$$
\mathcal{F}_\text{eff}^{\text{R}} = \mathcal{
$$

 $40(6, 7)$: $60, 6$

