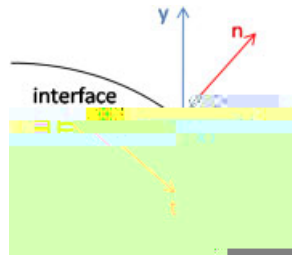


FIGURE 1. Geometric domain and its sub-domains.



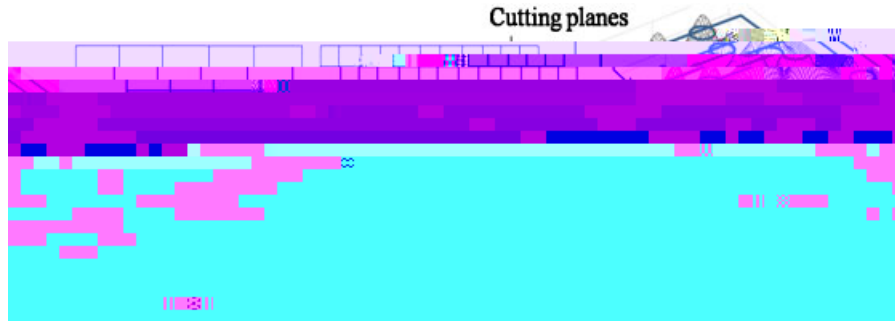


FIGURE 3. (a) $E_{\text{fac}}(d) = \dots$; (b) $E_{\text{fac}}(d) = \dots$; (c) \dots

$$N_I(\cdot) = \begin{cases} 0 & \text{if } \dots \\ 0 & \text{if } \dots \end{cases} \quad (17)$$

... $N_I(\cdot)$... $H(\cdot)$... $\chi(\cdot)$... $\phi(\cdot)$... $\nabla\phi(x)$...

$$(\cdot) = \frac{\nabla\phi(x)}{\|\nabla\phi(x)\|} \quad (18)$$

... $H(\phi)$... $\chi_j(\cdot) = |\phi(\cdot)| - |\phi(\cdot_j)|$...

$$H(\phi) = \begin{cases} 1 & \text{if } \dots \\ 0 & \text{if } \dots \end{cases} \quad \text{and } \chi_j(\cdot) = |\phi(\cdot)| - |\phi(\cdot_j)| \quad (19)$$

... $\chi_j(\cdot)$... χ_a ... χ_c ... χ_e ... χ_f ... χ_g ... χ_h ... χ_i ... χ_j ... χ_k ... χ_l ... χ_m ... χ_n ... χ_o ... χ_p ... χ_q ... χ_r ... χ_s ... χ_t ... χ_u ... χ_v ... χ_w ... χ_x ... χ_y ... χ_z ...

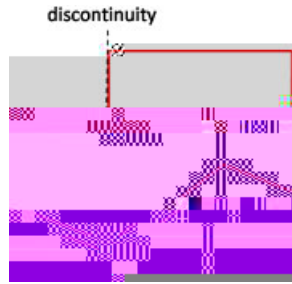


FIGURE 4. A schematic of a layered material with a surface discontinuity. (a) shows a layered material with a surface discontinuity. (b) shows a layered material with a surface discontinuity.

Bulk energy

So the bulk energy is given by $\delta \tilde{W}_b^e = \int_{\Omega} \delta \epsilon^e : \epsilon^e d\Omega = \delta \epsilon^T \cdot \int_{\Omega} \epsilon^T \{ \epsilon^e \} d\Omega$.

$$\delta \tilde{W}_b^e = \int_{\Omega} \delta \epsilon^e : \epsilon^e d\Omega = \delta \epsilon^T \cdot \int_{\Omega} \epsilon^T \{ \epsilon^e \} d\Omega \quad (20)$$

where ϵ^e is the strain tensor, ϵ^T is the transpose of the strain tensor, and $\int_{\Omega} \epsilon^T \{ \epsilon^e \} d\Omega$ is the volume integral of the product of the strain tensor and its transpose over the volume Ω .

External energy

External energy functional \tilde{W}_e is defined as follows:

$$\delta \tilde{W}_e = \delta \int_{\Omega} e^T d\Omega + \delta \int_{\partial\Omega_F} e^T d\Gamma \quad (30)$$

Final XFEM equation

Using the constitutive law (20), (22), (28), and (30), and the XFEM formulation (14) and (15), the final XFEM equation is:

$$\left(\frac{e}{b} + \frac{e}{d} + \frac{e}{s} \right) \cdot e = e - e_s \quad (31)$$

where e is the total strain, e_s is the surface strain, and e_b , e_d , and e_s are the strains associated with the bulk, the damage zone, and the surface, respectively.

$$e = [\nabla \cdot \mathbf{u}] \in \mathbb{R}^d$$

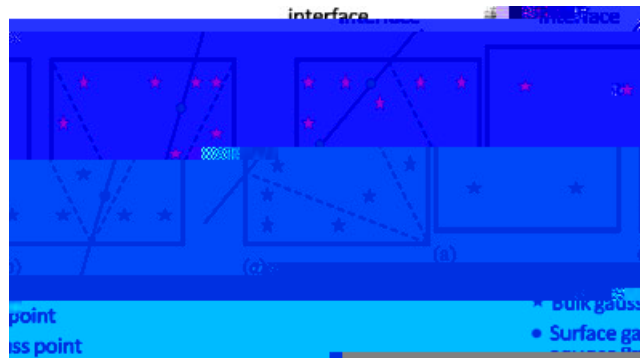


FIGURE 5. The construction of the scheme: (a) the bulk Gauss points; (b) and (c) the surface Gauss points.

Consider a domain Ω with an interface Γ separating two regions Ω_1 and Ω_2 . Let \mathbf{u} be a function defined on Ω . We consider the numerical approximation of \mathbf{u} using a finite difference method. The interface Γ is approximated by a set of points Γ_h . The bulk Gauss points are located in Ω_1 and Ω_2 , and the surface Gauss points are located on Γ_h . The numerical scheme is constructed by approximating the function \mathbf{u} at the bulk Gauss points using the function values at the surface Gauss points. The accuracy of the scheme is analyzed in [34].

NUMERICAL EXAMPLES

In this section, we present numerical examples of the proposed scheme. The results are compared with the exact solution. The numerical scheme is applied to a two-dimensional problem with an interface. The interface is approximated by a set of points. The numerical scheme is constructed by approximating the function at the bulk Gauss points using the function values at the surface Gauss points.

- The numerical scheme is applied to a two-dimensional problem with an interface. The interface is approximated by a set of points. The numerical scheme is constructed by approximating the function at the bulk Gauss points using the function values at the surface Gauss points.
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For the numerical examples, we consider a two-dimensional domain Ω with an interface Γ . The interface is approximated by a set of points Γ_h . The numerical scheme is constructed by approximating the function at the bulk Gauss points using the function values at the surface Gauss points.

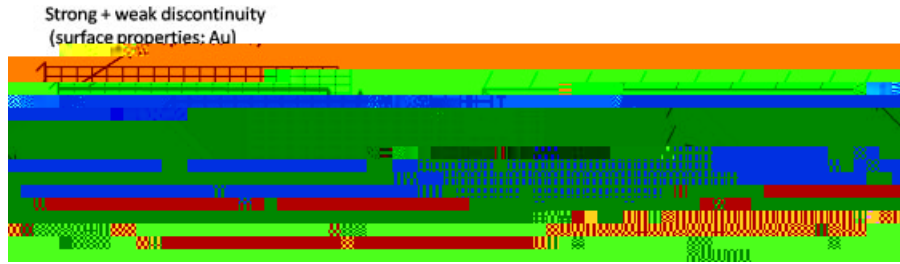


FIGURE 6. Schematic of the layered structure used in the XFEM analysis.

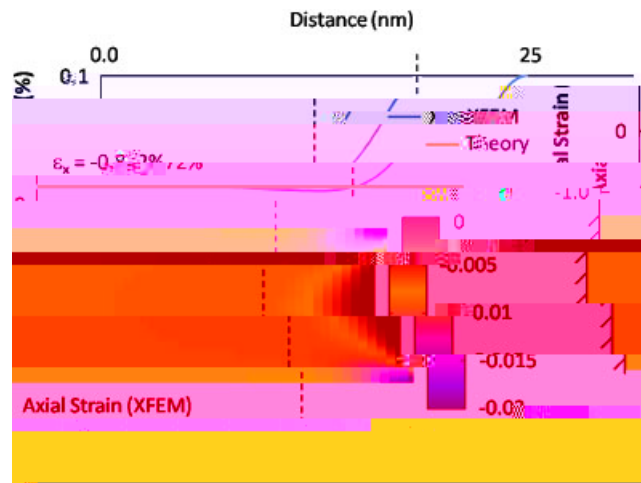


FIGURE 7. Axial strain distribution along the crack plane for the layered structure. The theoretical strain is given by $\epsilon_x = -0.01 \delta(x - 10)$ and the XFEM results are compared with the theoretical results.

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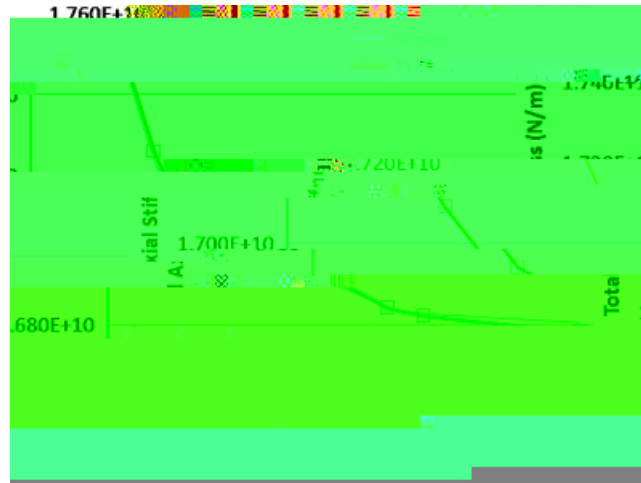


Fig. 9. Stress distribution in a crack tip.

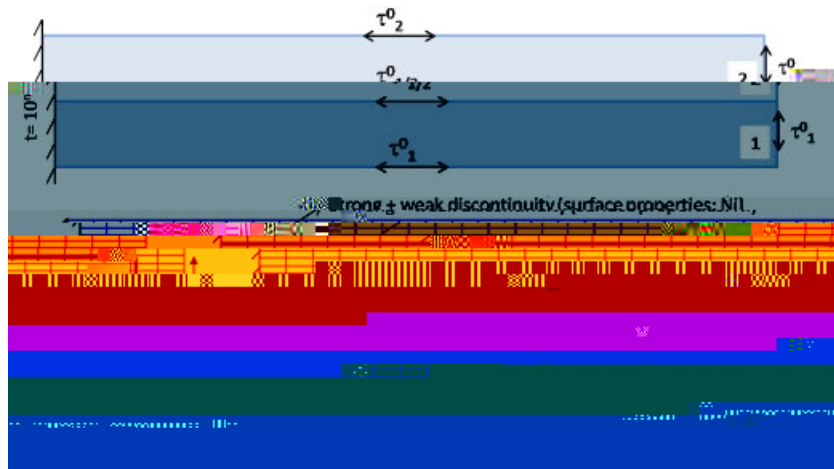


Fig. 10. Geometric and material properties of a layered material.

If $\tau_1^0, \tau_2^0, \tau_3^0$ and τ_{12}^0 are the initial stresses in the layers 1, 2 and 3 respectively, then the total stress τ in the material is given by:

$$M = (\tau_1^0)(t-x) - (\tau_2^0)x + (\tau_{12}^0) \frac{t}{2} - x \quad (47)$$

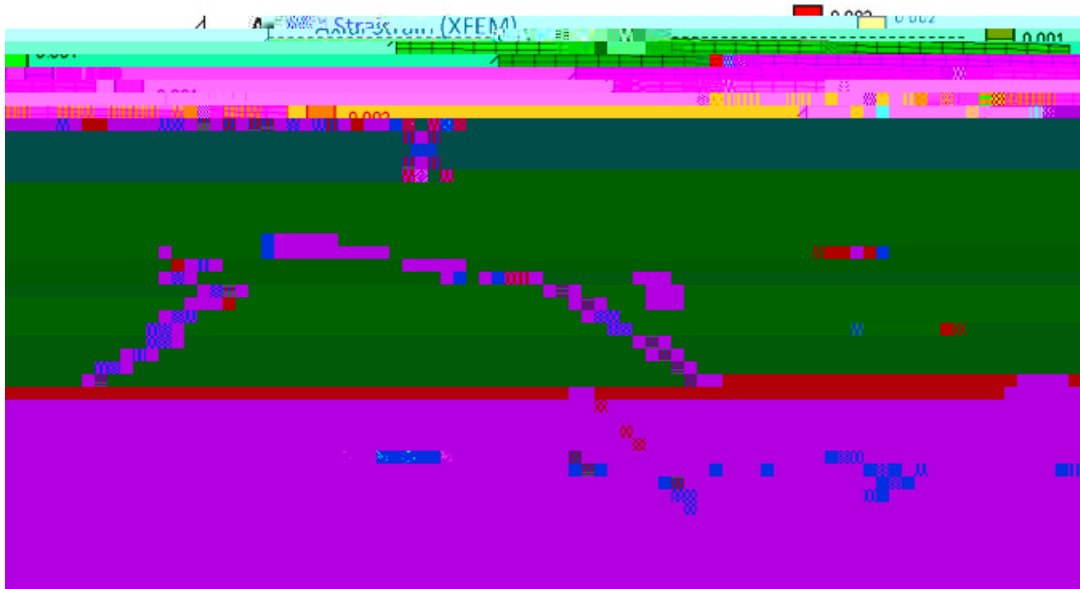


Figure 11. (a) A finite element analysis (XFEM) plot showing stress distribution in a layered material. The plot is titled "Strain (XFEM)". It features a color-coded stress field with a legend at the top right showing values from 0.000 to 0.002. The material is divided into several horizontal layers of different colors (green, blue, red, purple). A crack is visible, propagating through the layers. The stress concentration is highest at the crack tip.

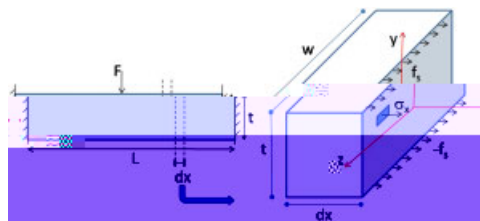


Figure 12. Schematic diagram of a beam under a point load F.

Figure 12 shows a beam of length L and thickness t under a point load F . A differential element of length dx is shown. The internal normal stress σ_x and shear stress f_s are indicated. The diagram illustrates the equilibrium of forces on the differential element.

$$\int_A \sigma_x y dA + 2 \int_0^t f_s \frac{t}{2} dz = -M \quad (53)$$

The normal stress σ_x is assumed to be linear across the thickness of the beam. The shear stress f_s is assumed to be constant across the thickness. The internal normal force is $\int_A \sigma_x y dA = S_{1111} \sigma_x$, where S_{1111} is the second moment of area. The internal shear force is $2 \int_0^t f_s \frac{t}{2} dz = 2 f_s t$. The equilibrium equation is $S_{1111} \sigma_x + 2 f_s t = -M$. The normal stress is $\sigma_x = E \epsilon_x$.

$= (y/a) \dots (53) \dots$

$$\frac{2E}{t} \int_A y^2 dA + S_{1111} t \dots = -M \tag{54}$$

Knowing that $I = \int y^2 dA$, ...

$$a = \frac{-M}{\frac{2EI}{t} + S_{1111} t} \tag{55}$$

For $t \ll \dots$, $I = \frac{1}{12} t^3$; ...

$$a = \frac{-M}{t(\frac{1}{6}Et + S_{1111})} \tag{56}$$

... $f \dots$

$$= \frac{-2My}{t^2(\frac{1}{6}Et + S_{1111})} \tag{57}$$

If \dots

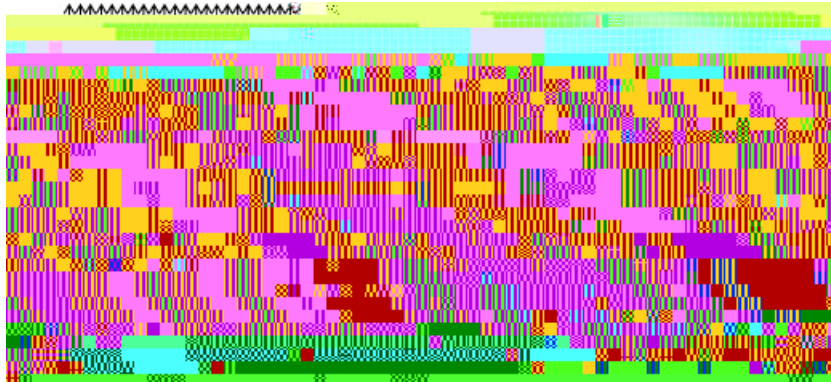


Figure 14. Geometric model of a nano-rod: (a) rod fixed at one end and (b) rod fixed at both ends. Sub-figure (c) shows the surface area of the rod.

