

with numerical aperture NA= 1.4 (Olympus). The vertical

Since H is quadratic in

of the relative ± 1 hedgehog topological charge of the point computer-simulated counterparts (Fig. 2), we can use detailed defect. When the defect is inspected from the same viewing cross-sectional presentations of the computer-simulated 3D direction (e.g., along the positive z -axis direction), the change director field to examine twisting within these structures of colors around two defects is found to be in the clockwise (Fig. 3). For this purpose, we choose a series of vertical and direction while that of the other two is in the counterclockwise horizontal cross-sectional planes that reveal the salient details direction. A comparison of directionality between defects for of the twist configuration. Consistent with experiments, the a given viewing angle indicates that the relative net total simulated structure reveals four hyperbolic point defects. Two charge of the point defects is zero, as is required by charge of them are located close to one of the confining surfaces, conservation to embed in the uniform homeotropic background while the other two are in the middle of a stretched torus of of the cell. Upon reversal of the viewing direction to be twist (Fig. 3). The twisted region is comprised of a looped along the negative z -axis, the direction of the color wheel double-twist tube that is stretched in one lateral direction. The reverses too, consistent with the LCOs nonpolar symmetry and depth location of the axis of the double-twist tube changes the fact that only relative charges ± 1 can be assigned to the when circumnavigating it along the closed extended loop it nematic point defects [1]. The family of iso-surfaces with forms, with its locations shifted upwards with respect to the different constant values of the z -component of $\mathbf{n}(\mathbf{r})$ [4] reveals structural details in addition to the topology information. The color pattern changes in accord with the continuous changes of the z -component of $\mathbf{n}(\mathbf{r})$ (n_z) and reveals a twist of the director field along different spatial directions, albeit, quantification of this director twisting is difficult to achieve with information from such surfaces alone. However, since the color-coded patterns of $\mathbf{n}(\mathbf{r})$ on all examined isosurfaces closely match the

isotropic handedness close to that of the equilibrium-state
chiral LC q

twisted localized structure to that of a uniform far-field background. Although the example provided here involves a toron with the net total twist of π from the central axis to periphery, with the double-twist torus of the twistion that has a Hopf index of zero, similar studies can be extended to torons and twistions with double-twist tori of a larger amount of twist and nonzero Hopf charge [6] that also possess hyperbolic defects, which will be described elsewhere. Beyond twistions and torons, we have observed similar types of behavior for different hopfions, cholesteric π -ngers, and π -nger loops, as well as for other localized structures in confined cholesteric LCs. We will discuss these other localized structures elsewhere.

We have demonstrated previously that torons can correspond to both equilibrium (global free energy minimum) and metastable (local free energy minima) states of confined cholesteric LCs [3], depending on material and geometric parameters such as the cell thickness to pitch ratio, elastic constants, and so on [3].

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